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HEAT TRANSFER ANALYSIS FOR UNSTEADY HIGH VELOCITY PIPE FLOW

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U.S. ARMY ARMAMENT COMMAND

FINAL REPORT

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Contract No. DAA-G29-76-G-0123

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(18) RIA-R

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(19) **REPORT DOCUMENTATION PAGE**

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER R-CR-77-019	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER rept.
4. TITLE (and Subtitle) (6) Heat Transfer Analysis for Unsteady High Velocity Pipe Flow,		5. TYPE OF REPORT & PERIOD COVERED (9) Final (Jan. 1976-June 1976)
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Ching Jen Chen, Peter Li, Jenq Shing Chiou, Hsai Yin Lee		8. CONTRACT OR GRANT NUMBER(s) (15) DAA-G29-76-G-0123
9. PERFORMING ORGANIZATION NAME AND ADDRESS Institute of Hydraulic Research The University of Iowa Iowa City, Iowa. 52242		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Research Office Box CM Duke Station Durham, North Carolina 22706		12. REPORT DATE (11) April 1977
		13. NUMBER OF PAGES 109
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) CMDR, Rock Island Arsenal GEN Thomas J. Rodman Laboratory, SARRI-RLR Rock Island, Illinois 61201		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

(12) 116p.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Heat Conduction, Error Analysis, Gun Barrel

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report consists of three parts. The first is the analysis for minimization of the temperature distortion due to the thermocouple cavity. The error is minimized or reduced to zero by optimizing the combination of cavity diameter and depth and the thermocouple transport properties and size. The second is the refinement of the presently available computer program for prediction of the surface temperature and heat flux at the next page

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S/N 0102-014-6601

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June 1976

Final Report (Jan. 1, 1976 - June 1976)

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ABSTRACT

This report consists of three parts. The first is the analysis for minimization of the temperature distortion due to the thermocouple cavity. The error is minimized or reduced to zero by optimizing the combination of cavity diameter and depth and the thermocouple transport properties and size. The second is the refinement of the presently available computer program for prediction of the surface temperature and heat flux at the inner surface of the pipe by inverting the temperature response measured by an interior probe close to the heated surface. The refinement is achieved by using the double precision format in the program and adapting the dimensionless formulation. The third is to study the inversion solution for a large time duration of a time dependent surface heat flux. The solution is obtained by the method of Laplace transformation with the convolution integral.

FOREWORD

This is the final report of the project entitled "Heat Transfer Analysis for Unsteady High Velocity Pipe Flow." The project was funded by Army Research Office Grant DAA-G29-76-G-0123 for a six month period from January 1976 to June 1976.

The work done for project is reported here in three parts. The first is the analysis for minimization of the temperature distortion due to the thermocouple cavity. The error is minimized or reduced to zero by optimizing a combination of cavity diameter and depth and the thermocouple material and size. The second is the refinement of the presently available computer programs for prediction of the surface temperature and heat flux at the inner surface of the pipe by inverting the temperature measured by an interior probe close the the heated surface. The refinement is achieved by using the double precision format in the program and adapting the dimensionless formulation. The third is to study the inversion solution for a large time duration of a time dependent surface heat flux. The solution is obtained by the method of Laplace transform and the convolution integral. Each of the above three subjects is reported as a part of the present report.

The investigator would like to thank Mr. D. M. Thomsen of General Rodman Laboratory, Rock Island Arsenal, Rock Island, Illinois for his constant participation in the course of the research.

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NOMENCLATURE

c	specific heat
d_t	diameter of thermocouple
d	diameter of cavity
D	thickness of the disk
Q	constant heat flux
T	temperature
t	real time
x	X/D
X, Y	coordinate along and normal to the heated surface
y	Y/D
Error %	[θ (at cavity base) - θ (at edge of disk)] x 100, or distorted temperature divided by QD/κ_1

Greek letters

ε	distance between the base of cavity to the heated surface
θ	dimensionless temperature $T\kappa_1/QD$
τ	dimensionless time
α	thermal diffusivity
κ	thermal conductivity
ρ	density

subscripts

1	the disk material
2	insulation material
3	thermocouple material
t	thermocouple

PART I

ON MINIMIZATION OF TEMPERATURE DISTORTION IN THE THERMOCOUPLE CAVITY

SUMMARY - PART I

When a thermocouple is embedded in a solid to measure temperature a distortion of temperature is created because of the thermocouple cavity. For a given solid under measurement the temperature distortion at the base of the thermocouple cavity can be minimized by a proper choice of the thermocouple size and its material, the cavity diameter and the depth of the cavity. This optimum combination is solved in this study by a finite element analysis for the case that the thermocouple cavity is drilled into the center of a disk. The disk, initially at a uniform temperature, is then heated with a constant heat flux on one suface and is insulated on the other surface. The calculated result covers a range of thermal conductivity and diffusivity for most commonly used thermocouples. For ease in practical applications a simple formula for determining the optimum ratio of the thermocouple diameter to that of the cavity is given as a function of density - specific heat product and thermal conductivity of solid material, thermocouple, and insulating material.

LIST OF FIGURES - PART I

LIST OF TABLES - PART I

INTRODUCTION

A direct measurement of transient surface temperature and heat flux is often difficult. For example, a surface involves two modes of heat transfer, say, radiative and convective heat transfer. In this case if the measuring probe has a different radiative property than that of the surface, erroneous measurements will result. A piston or projectile sliding over the cylindrical surface is another case where the direct measurement at the surface is difficult. A surface involving melting or ablation is also difficult to make direct measurement. Therefore, indirect estimation by inverting the temperature history inside the heat conducting solid as measured by a thermocouple is often used for prediction of the surface temperature and heat flux. Beck [1], Hernning and Parker [2], Frank [3], Imber and Khan [4], Stolz [5], Chen and Thomsen [6] have developed different inversion solutions for this purpose. All of these solutions assumed that the cavity drilled into the solid does not distort the true temperature distribution. Thus, it is important that the temperature measurement by an interior probe is accurate and involves least distortion or error. Theoretically Beck [7], Masters and Stein [8], Burnett [9], and Chen and Li [10] studied the distortion of the temperature field in the presence of a thermocouple and its cavity. Experimentally Chen and Danh [11] showed that appreciable distortion, say 10%, of temperature field may exist for a normal implant of the thermocouple into a solid body. From studies of Chen and Li [10] and Beck [7] they found that with a proper combination of the thermocouple cavity diameter, its depth, and the thermocouple material and its diameter,

the distortion of temperature field with respect to space or time can be minimized if not eliminated. In this report we study the optimum combination of these parameters such that at a given situation one knows what is the best combination to use and what is the magnitude of the temperature distortion.

FORMULATION OF PROBLEM

In the present study we consider a disk depicted in Figure 1 which has a thickness D and is drilled a cavity of a diameter to a depth of ε distance from the heating surface. The heat flux Q is assumed to be constant and the upper surface of the disk is assumed to be insulated. A thermocouple of a diameter d_t is then welded on the cavity base. Furthermore, the disk may be thought to approximate a cut out from a hollow cylinder if the radius of the disk is small compared with the radius of the cylinder. The diameter of the disk is choosen to be D where the temperature distortion due to the thermocouple cavity becomes negligible. For this to be true one needs to restrict the ratio of cavity diameter to the disk diameter d/2D be small. The unfilled cavity can be air or insulating material.

The basic idea to minimize or to eliminate the temperature distortion is based on a proper choice of the thermocouple size and the material which has a higher thermal conductivity than that of the disk so as to conduct more heat away at the cavity base balancing the insulation effect of the insulator in the cavity.

Let X and Y be respectively the coordinate along and normal to the heated disk surface and the Y axis coincide with the axis of the cavity.

The thermal conductivity is assumed to be constant and the temperature distribution is axisymmetrical. The governing equations for the transient heat conduction in dimensionless form are:

for the disk (subscript "1", see Figure 2)

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial x^2} + \frac{1}{x}\frac{\partial \theta}{\partial x} + \frac{\partial^2 \theta}{\partial y^2} \tag{1}$$

for the insulating material in the cavity (subscript "2")

$$\frac{\partial \theta}{\partial \tau} = \frac{\alpha_2}{\alpha_1}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{1}{x}\frac{\partial \theta}{\partial x} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{2}$$

for the thermocouple (subscript "3")

$$\frac{\partial \theta}{\partial \tau} = \frac{\alpha_3}{\alpha_1}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{1}{x}\frac{\partial \theta}{\partial x} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{3}$$

where $\tau = \alpha_1 t/D^2$ is the dimensionless time, $x = X/D$ the dimensionless radial coordinate, $y = Y/D$, the dimensionless distance normal to the heated surface. α_1, α_2, and α_3 are respectively the thermal diffusivity of the disk, insulating material, and the thermocouple. The dimensionless temperature θ is defined as $T\kappa_1/QD$ where T is the temperature above the initial, uniform temperature.

The intial temperature of the disk is then

$$\theta\ (x, y, 0) = 0 \tag{4}$$

The boundary conditions, see Figure 2, are:

a constant heat flux at the lower surface,

$$y = 0 \qquad \left.\frac{\partial\theta}{\partial y}\right|_{y=0} = 1 \tag{5}$$

the insulation at the upper surface,

$$y = 0 \qquad \left.\frac{\partial\theta}{\partial y}\right|_{y=0} = 0 \tag{6}$$

the zero temperature distortion at the edge of the disk,

$$x = 1 \qquad \left.\frac{\partial\theta}{\partial x}\right|_{x=1} = 0 \tag{7}$$

and the axisymmetric condition at the cavity axis

$$x = 0 \qquad \left.\frac{\partial\theta}{\partial x}\right|_{x=0} = 0 \tag{8}$$

The condition (7) of the zero temperature distortion was verified by Chen and Li [10] in their early calculation when the cavity diameter is one tenth of the disk diameter. In addition to the above boundary condition the temperature and heat flux at the interface of the disk, thermocouple, and insulating material are taken to be continuous.

ANALYSIS

There are five parameters that can be varied for the present analysis. They are (a) the dimensionless distance from the base of the cavity to the heated surface ε/D, (b) the size of the cavity d/D, (c) the ratio of the thermocouple diameter to that of the cavity, d_t/d.

(d) the thermal conductivity ratio κ_2/κ_1, and κ_3/κ_1 which comes from the continuity of heat flux at interfaces. (e) the ratio of the product of density and specific heat $\rho_3 c_3/\rho_1 c_1$ (or equivalently to the ratio of of thermal diffisivity α_3/α_2 or α_2/α_1). The subscripts 1, 2 and 3 denote the disk, insulation and thermocouple materials.

Because of the complexity of the geometry and the multiplicity of material the method of finite element technique as discussed by Wilson and Nikel [12] is adapted with the aid of a computer program developed by Wilson [13]. The present problem is subdivided into finite element as required by the method, see Figure 2. The dimensionless pie section is subdivided into 121 finite elements with 12 dividing lines on both coordinates. Each element is defined by four nodal points where nodal points are denoted by intersections of the dividing lines and numbered as shown in Figure 2. The material property corresponding to each element is then assigned to the program developed by Wilson [13]. The solution at each nodal with respect to time is then obtained.

For numerical calculation three typical values of the distance from heated surface to the base of the cavity ε/D are chosen to be 0.04, 0.1 and 0.2. The cavity diameter is fixed at one tenth of the disk diameter. The thermocouple to cavity diameter ratio d_t/d is made to vary 0, 0.2, 0.4, 0.6, 0.8 and 1.0. Regarding the range of the ratio of the thermal conductivity and the ratio of the product of density and specific heat we surveyed these ratios for the commonly used thermocouples in Table 1 [14] and plotted in Figure 3. Figure 3b shows the ratio of

thermocouple conductivity to that of the disk κ_3/κ_1 versus the ratio of density - specific heat product ρ_3c_3/ρ_1c_1 where the value of the conductivity is taken to be the average value between 200 and 800°K. One sees that for most practical situations the ratio of density - specific heat product ρ_3c_3/ρ_1c_1 is approximately constant at 0.7 except when the conductivity ratio κ_3/κ_1 is small. Thus the value of ρ_3c_3/ρ_1c_1 and κ_3/κ_1 for calculation are chosen, as shown in triangular symbols of Figure 3b, to cover the practical range. The corresponding value for ρ_2c_2/ρ_1c_1 and κ_2/κ_1 for the insulation material are chosen to be fixed at 0.5 and 0.005 which is a typical value for Teflon insulating material and is also approximately the order of magnitude for air.

RESULTS AND DISCUSSIONS

Numerical results of the calculations are presented in Tables 2 to 4 and Figures 4 to 9. The percentage error of temperature is defined as the distorted temperature divided by a reference temperature defined by QD/κ_1. Tables 2 to 4 give the percentage error of temperature distortion) as a function of time for different values of the parameters d_t/d (0 to 1.0), κ_3/κ_1 (0.5 to 10), ρ_3c_3/ρ_1c_1 (0.5 to 1.8) and ε/D (0.02 to 0.1).

Figures 4, 5 and 6 show the three typical temperature distributions in the steel disk near the thermocouple junction for the case $\varepsilon/D = 0.06$ $d/2D = 0.1$ at the time $\tau = 0.08$. Figure 4 is the temperature distribution when the cavity is filled entirely with the insulation material (Teflon "2" $\kappa_2/\kappa_1 = 0.005$, $\rho_2c_2/\rho_1c_1 = 0.5$). The dimensionless isotherms

$T\kappa_1/QD$ shows the distortion of temperature distribution. One sees that the insulation effect on the heat transfer near the cavity base not only creates a much higher junction temperature of $T\kappa_1/QD = 0.342$ than the undistorted one of 0.255 at the edge of the disk giving an error of 8.7% but also causes a hot spot at the heating surface with a higher temperature of $T\kappa_1/QD = 0.374$ over the undistorted one of 0.311. On the other hand the temperature distribution in the insulation material is much lower than the true temperature creating a large temperature gradient at the base of the cavity. Figure 5, contrary to Figure 4, is the temperature distribution when the cavity is completely filled with thermocouple material whose thermal conductivity is ten times larger than the disk material e.g., copper versus steel. Now over-conduction of heat by the thermocouple has created a cold spot at the base of the cavity giving, $T\kappa_1/QD$ of 0.17 versus the undistorted one of 0.255 with an error of 8.5% as well as at the heating surface with $T\kappa_1/QD$ of 0.236 versus 0.311. The temperature distribution in the thermocouple now becomes higher than the undistorted one in the disk. By properly choosing the ratio of thermocouple diameter to that of the cavity one may minimize these distortions of the temperature response at the base of the cavity. This is shown in Figure 6 where the thermocouple diameter d_t is chosen to be 0.4 of the cavity diameter d with $\kappa_3/\kappa_1 = 10$, e.g. copper-steel combination. Figure 6 shows that distortions at the base of the cavity and at the heating surface are almost eliminated giving the temperature $T\kappa_1/QD$ of 0.245 and 0.320 with respectively the error of 1% and 0.9%.

In order to examine the details of the distorted temperature response at the base of the cavity we tabulated the results in Tables 2, 3 and 4 and plotted the error percentage as function of the ratio of the thermocouple diameter to that of the cavity for different cavity depth, time, and thermal conductivity in Figures 7, 8 and 9. In these Figures the ρc ratio ranges from 0.7 to 1.3 which covers most of the practical applications.

In the case when the ρc ratio is equal to or less than one the errors in temperature response at the base of the cavity in these figures are all positive or overheat for $\kappa_3/\kappa_1 \leq 1$. This is because the thermal conductivity of the thermocouple is less than that of the disk and the heat capacity ρc of the thermocouple is also small. Therefore, no extra conduction of heat can be achieved by the thermocouple to compensate for the blocking of the heat transfer by the insulation material in the cavity. On the other hand, if $\kappa_3/\kappa_1 > 1$ the error of temperature varies from positive value for $d_t/d = 0$ to some negative value as d_t/d approaches 1. Thus for $\kappa_3/\kappa_1 > 1$ a properly chosen combination of thermocouple and insulation material can minimize the error. For example, in Figure 7 a combination of $\kappa_3/\kappa_1 = 10$, $\rho_3 c_3/\rho_1 c_1 = 0.75$, $\varepsilon/D = 0.1$ and $d_t/d = 0.5$ produces almost negligible error. This combination shown to be optimum at $\tau = 1.0$ in Figure 7 is also optimum for other time periods (see Table 2). Therefore once an optimum combination of parameters is chosen it is valid throughout the entire transient period of an experiment. From Figures 7, 8 and 9 one can also see that the optimum ratio of d_t/d which gives zero temperature error decreases as the κ_3/κ_1 ratio increases.

This implies that for the thermocouple with a larger thermal conductivity a smaller diameter is sufficient to eliminate the temperature distortion. The result shown in Figures 7, 8 and 9 can in general be adapted for use in practical application to choose the size of thermocouple and cavity, the thermocouple material and the depth of the cavity to be drilled.

As mentioned earlier that when $\kappa_3/\kappa_1 \leq 1$ and $\rho_3 c_3/\rho_1 c_1 \leq 1$ the errors of the temperature response at the base of the cavity are all positive. However we found (see Tables 2.2, 3.2 and 4.2) that if $\rho_3 c_3/\rho_1 c_1$ ratio is made large enough during the transient period the error of temperature response at the base of the cavity may indeed become negative even when $\kappa_3/\kappa_1 \leq 1$. Physically although the thermal conductivity of the thermocouple κ_3 is smaller than that of the disk material, but with a larger heat capicitance $\rho_3 c_3/\rho_1 c_1 > 1$ the thermocouple is still capable of absorbing extra heat flux and hence eliminates the temperature distortion at the cavity base during the transient period. To illustrate this fact we examine Figure 7 (or see Table 4.2) for the data of $\kappa_3/\kappa_1 = 1$ and $\rho_3 c_3/\rho_1 c_1 = 1.3$. One sees that when $d_t/d = 1$ the temperature distortion can indeed be negative. Therefore, if d_t/d are choosen between 0.8 and 1 the error can be minimized. However one must keep in mind that the elimination of error by heat capacitance can work during the transient period only, for once a steady state conduction is established the heat capacity ρc will no longer have any effect and overheating at the cavity base eventually will develop. This can be seen best from the governing equation (1) that at steady state the unsteady term which contains ρc product is zero and is not a parameter affecting the distortion.

Another important fact that should be mentioned is that in general the optimum choice of d_t/d ratio for given κ_3/κ_1 and ρc ratio does not vary very much with the variation of ε/D ratio. The insensitivity of the optimum d_t/d ratio to the ε/D ratio ranging from 0.02 to 0.1 means that the distortion of the temperature is insensitive to the cavity depth or the thickness of the disk. This fact was already pointed out by Chen and Danh [11] in their experiment that the temperature distortion at the base of the cavity is more sensitive to the variation of the cavity diameter than the depth of the cavity drilled.

As an example of a practical application, let us consider a measurement of the transient temperature response of an engine block made of aluminum. From Figure 3b we know that aluminum has high thermal conductivity. Therefore copper-constantan thermocouple which has a higher thermal conductivity than aluminum should be chosen. For this material combination we have $\kappa_3/\kappa_1 = 1.69$ $\rho_3 c_3/\rho_1 c_1 = 1.3$. Now if the thermocouple cavity is drilled such that $\varepsilon/D = 0.1$ then from Figure 7 interpolating between $\kappa_3/\kappa_1 = 2$ and 1 for $\rho_3 c_3/\rho_1 c_1 = 1.3$ we find that the optimum d_t/d for $\kappa_3/\kappa_1 = 1.69$ is approximately 0.7

One disadvantage of invoking finite element analysis is that the result does not give a clear functional relation among the parameters involved. In an attempt to obtain a simple and useful relation to relate the various parameters we note the following fact and result: (a) the optimum d_t/d ratio for zero temperature distortion is a strong function of κ_3/κ_1 and ρc ratio but is relatively insensitive to the ε/D ratio, (b) from the theoretical reasoning the d_t/d ratio is independent

of ρc ratio if the problem is steady state. A simple steady one dimensional analysis in which the thermocouple and the insulation material in the cavity is made to conduct the same amount of heat that would be transferred without the cavity gives the relation

$$d_t/d = \sqrt{(\kappa_1 - \kappa_2)/(\kappa_3 - \kappa_2)} \qquad (9)$$

Using the above equation as a base we find that for the transient heat conduction as calculated by the finite element method the following equation (10) correlates very well with the optimum d_t/d ratio.

$$d_t/d = (\rho_3 c_3/\rho_1 c_1)^{0.3} \sqrt{(\kappa_1 - \kappa_2)/(\kappa_3 - \kappa_2)} \qquad (10)$$

Equation (10) gives an error or distortion of no more than two percentage points. In practice equation (10) may be used as a rule of thumb.

CONCLUSION

An analysis of the temperature distortion caused by the cavity drilled into a disk to accommodate the thermocouple has been studied. The calculation is carried out for the case of constant heat flux. It is shown that the temperature at the base of the cavity distorted from that without a cavity can be eliminated by a properly chosen combination of the ratio of the thermocouple diameter to the cavity diameter, d_t/d and the thermocouple material κ_3/κ_1. The optimum ratio of d_t/d can be found from Figures 7, 8 and 9 or Tables 2, 3 and 4, or approximately from equation (10). As a rule the thermocouple must be chosen

to have a higher thermal conductivity than that of the heat conducting solid. The cavity diameter should be as small as practically possible. For the case of time dependent surface heat flux the present result may be also used as a general guide.

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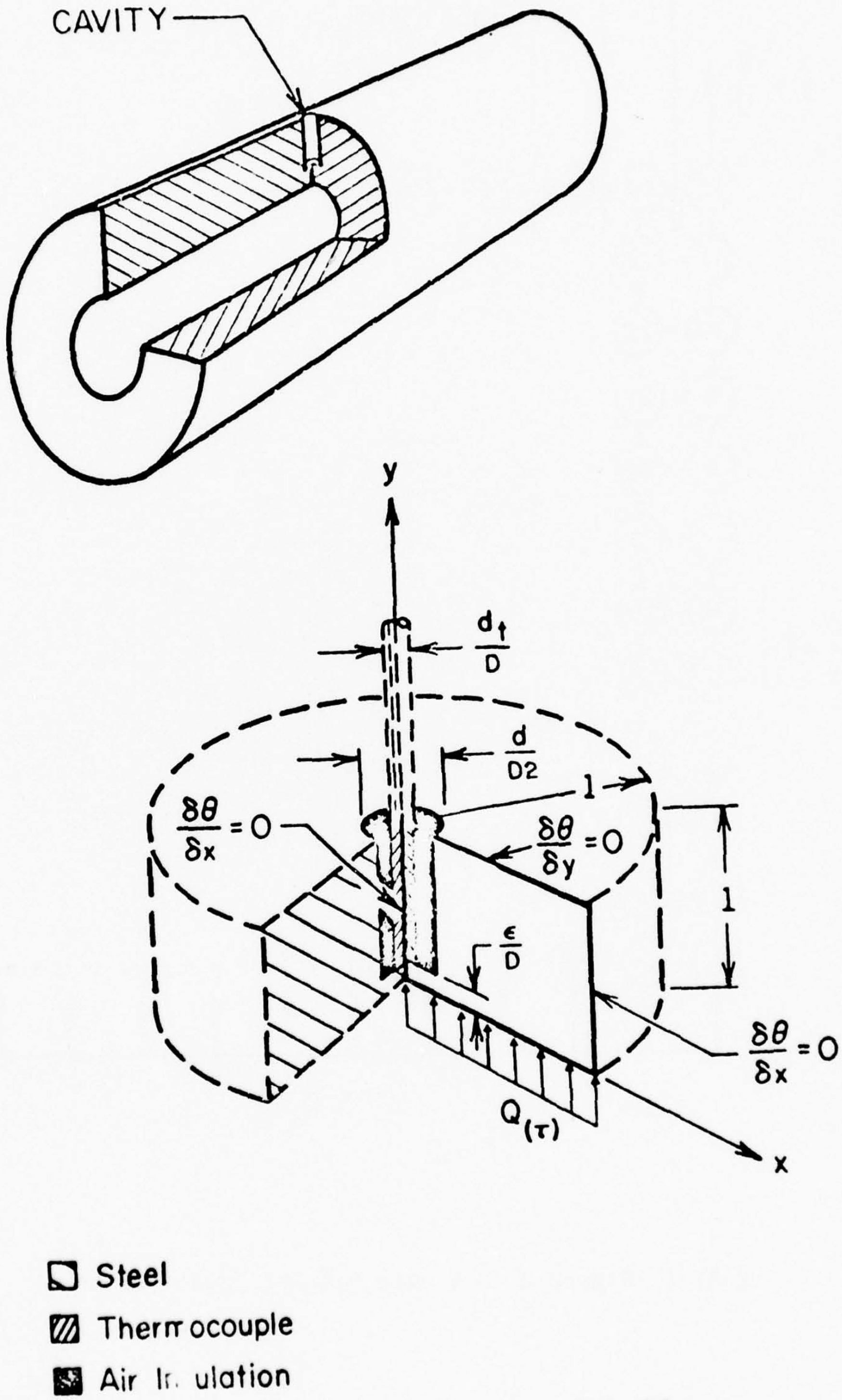
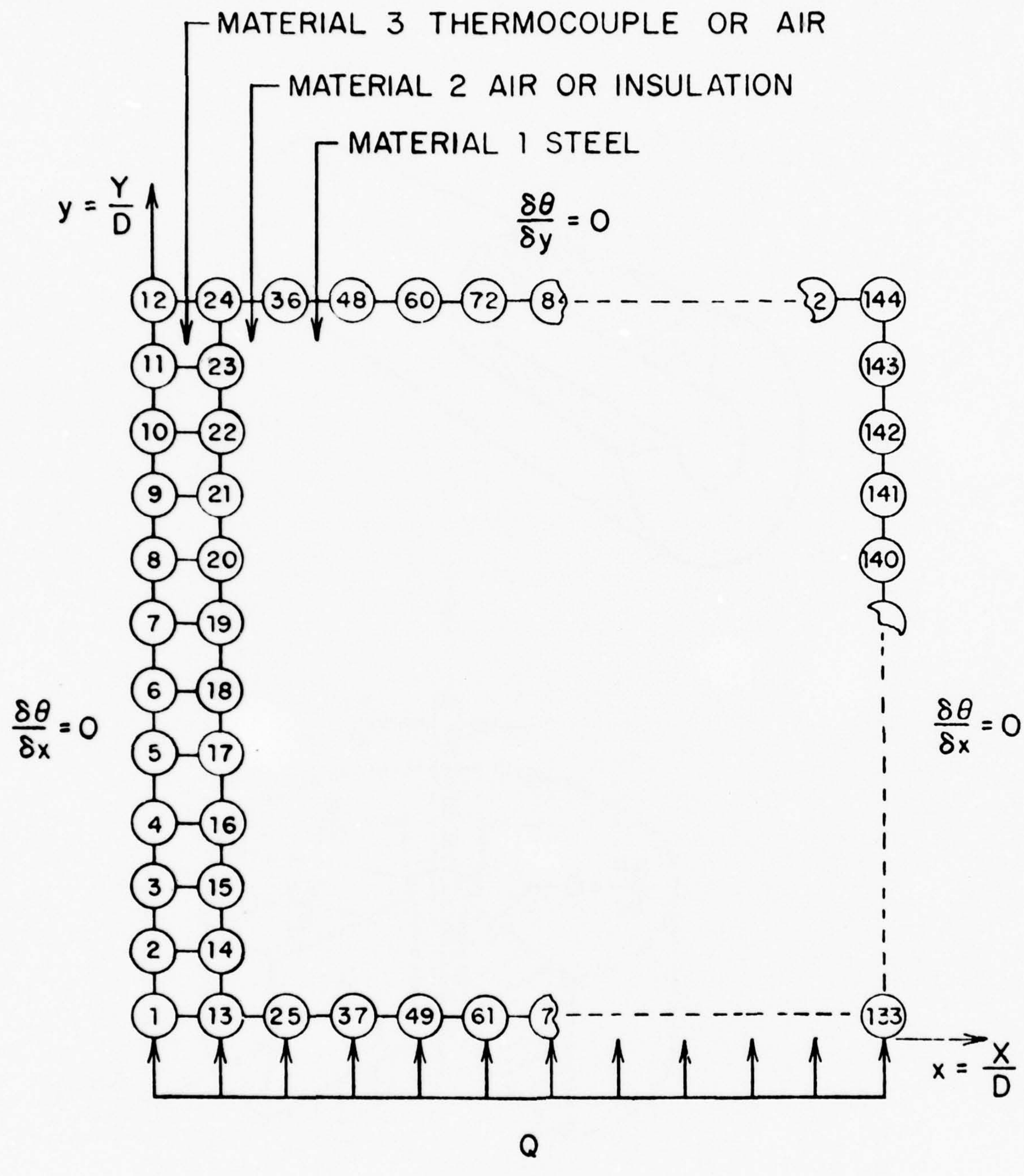
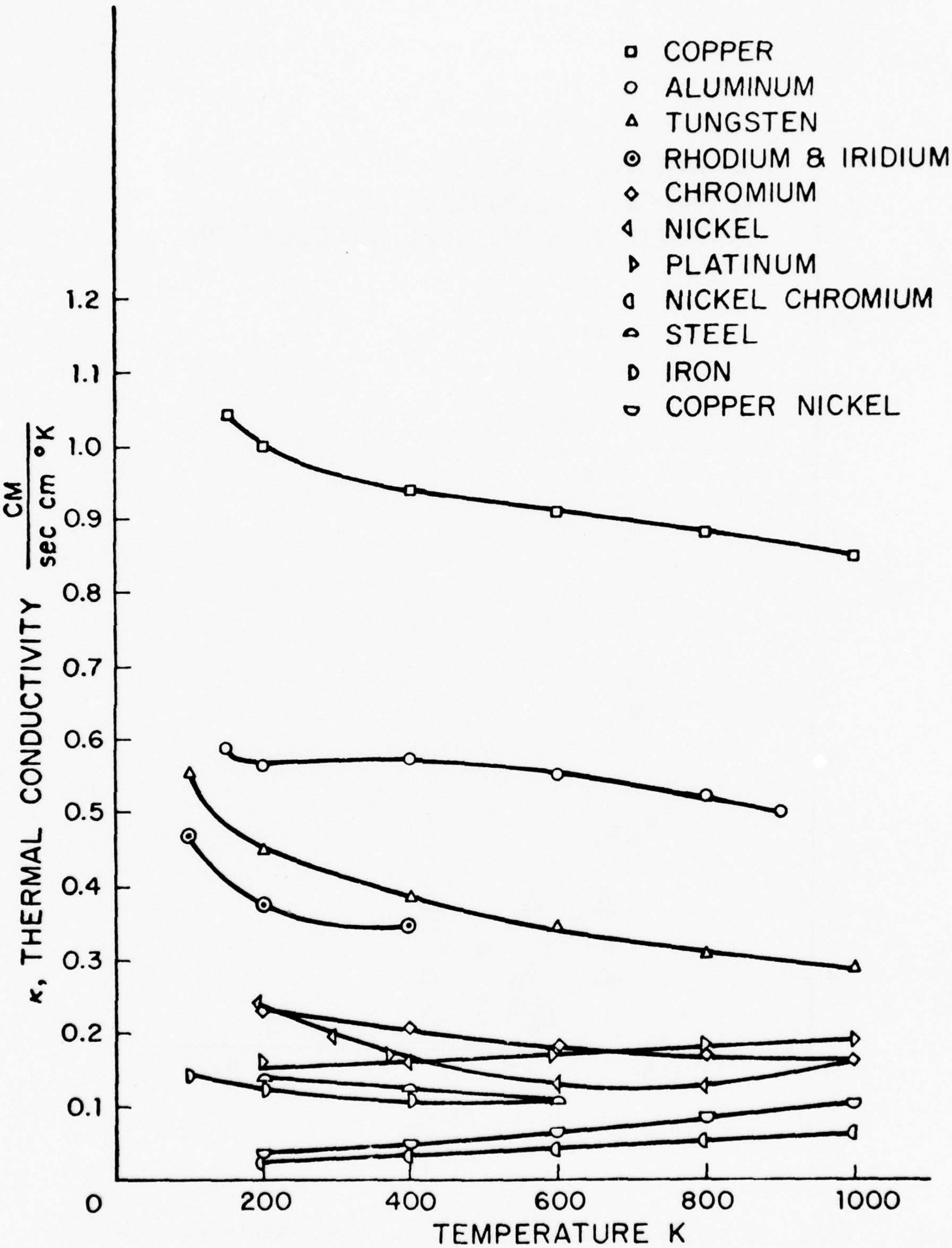


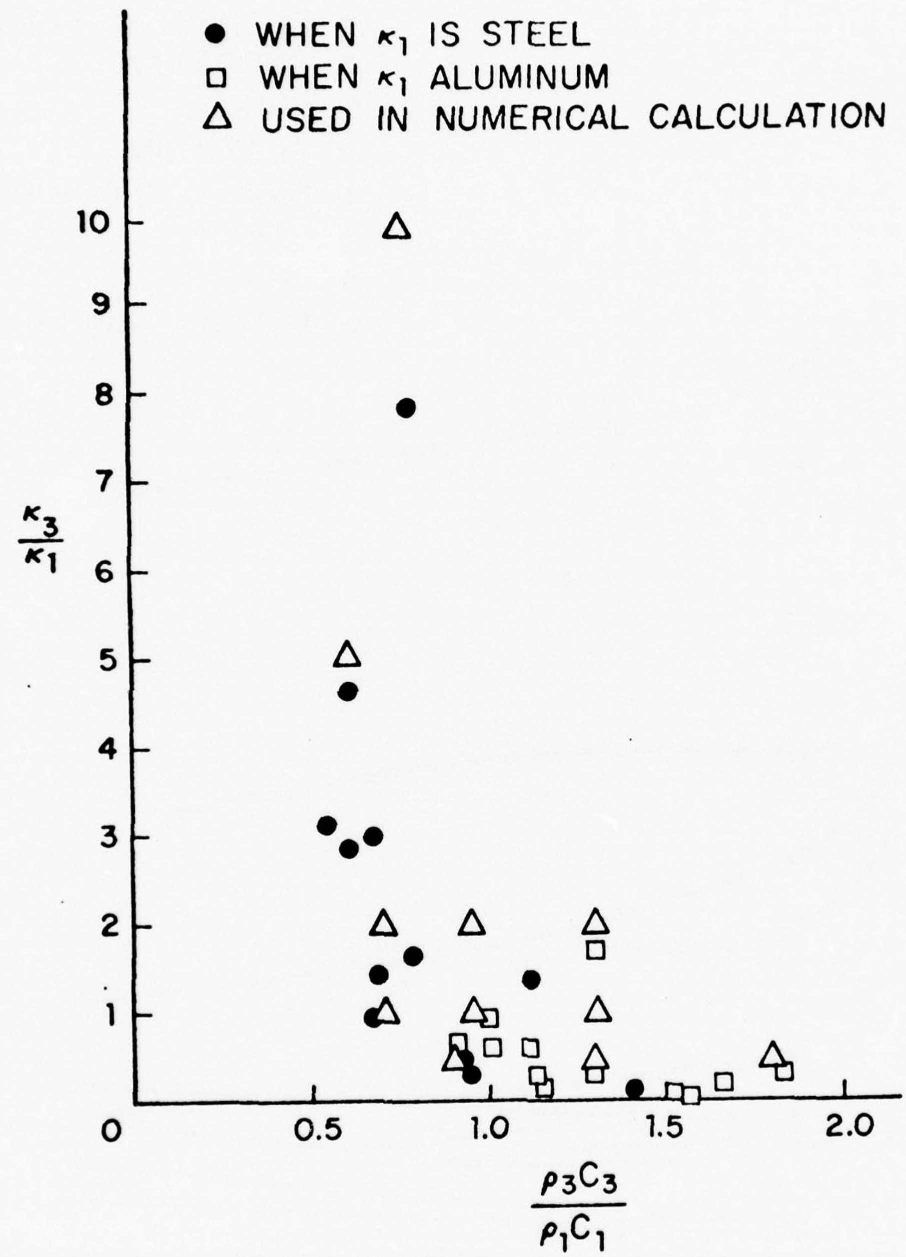
PART I Figure 1 Geometric Representation of Problem



PART I Figure 2 Finite Element Idealization

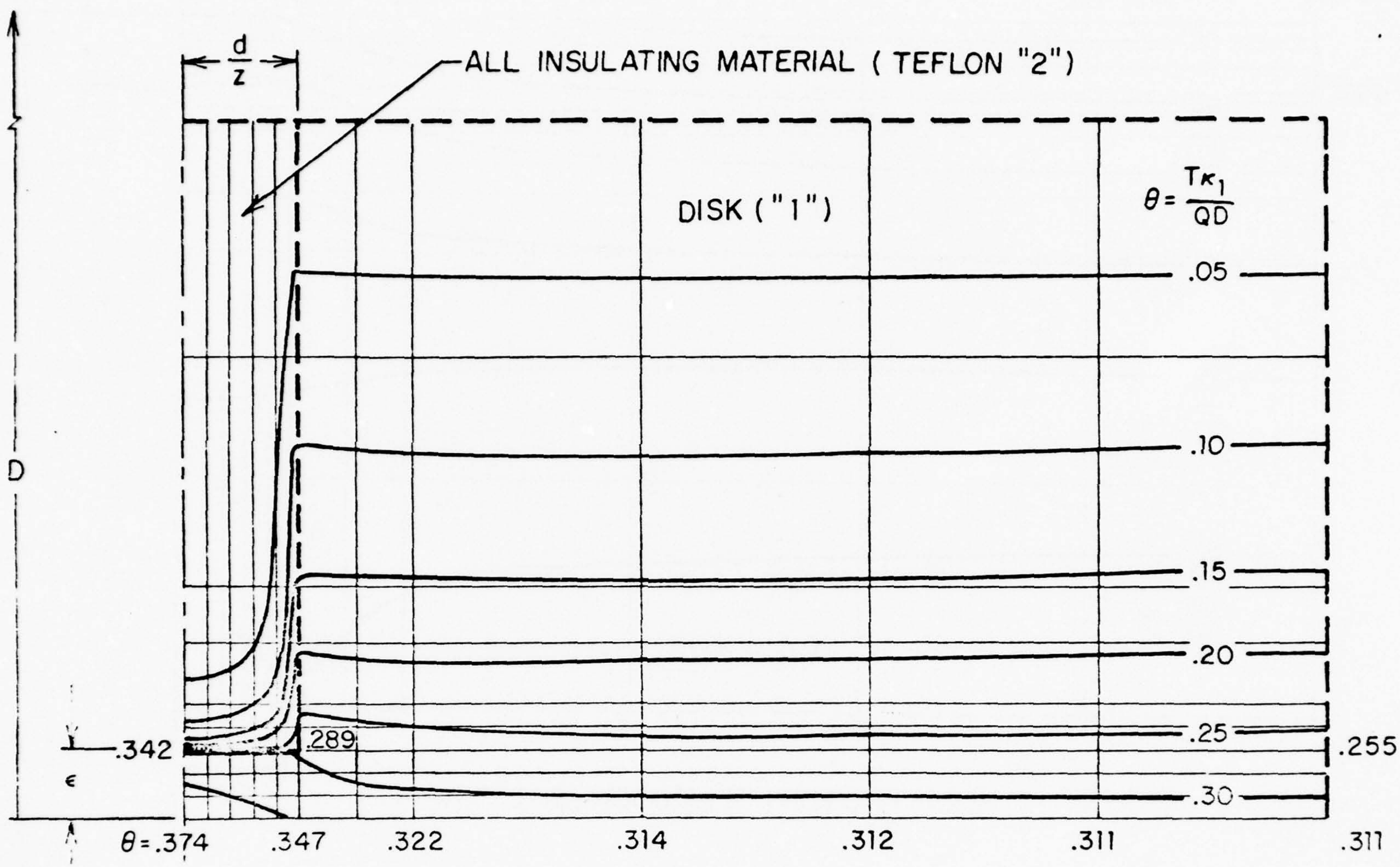


PART I Figure 3a Thermal C nductivity of Thermocouple Materials



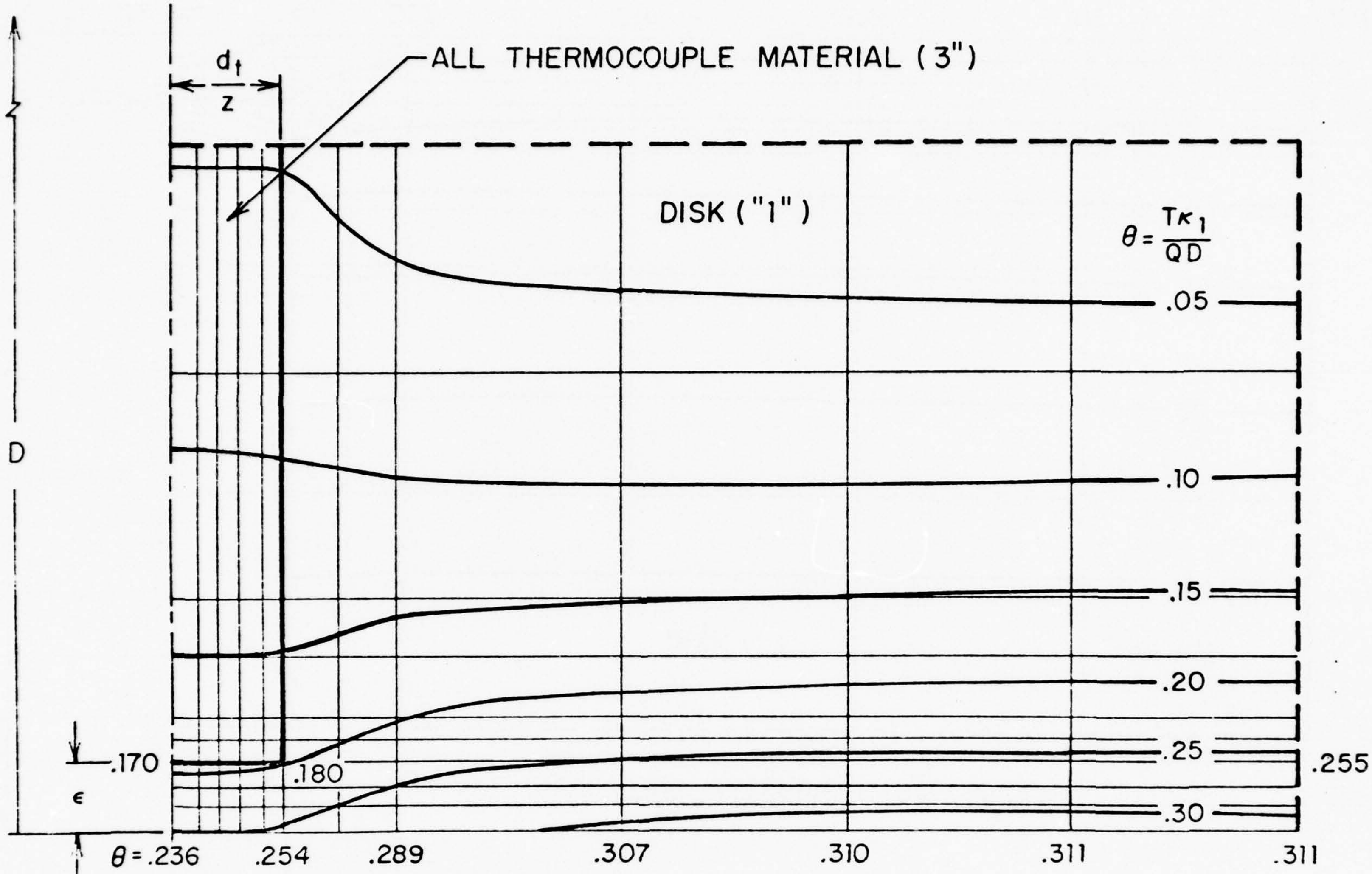
PART I Figure 3b Variation of Thermal Conductivity Ratio and Density - Specific Heat Ratio

$$\tau = .08,\ \frac{\kappa_2}{\kappa_1} = .005,\ \frac{\rho c_2}{\rho c_1} = .5,\ \frac{d_t}{d} = 0,\ \frac{\epsilon}{D} = .06,\ \frac{d}{2D} = .1$$



PART I Figure 4 Temperature Distribution with Insulation Material Filled the Cavity

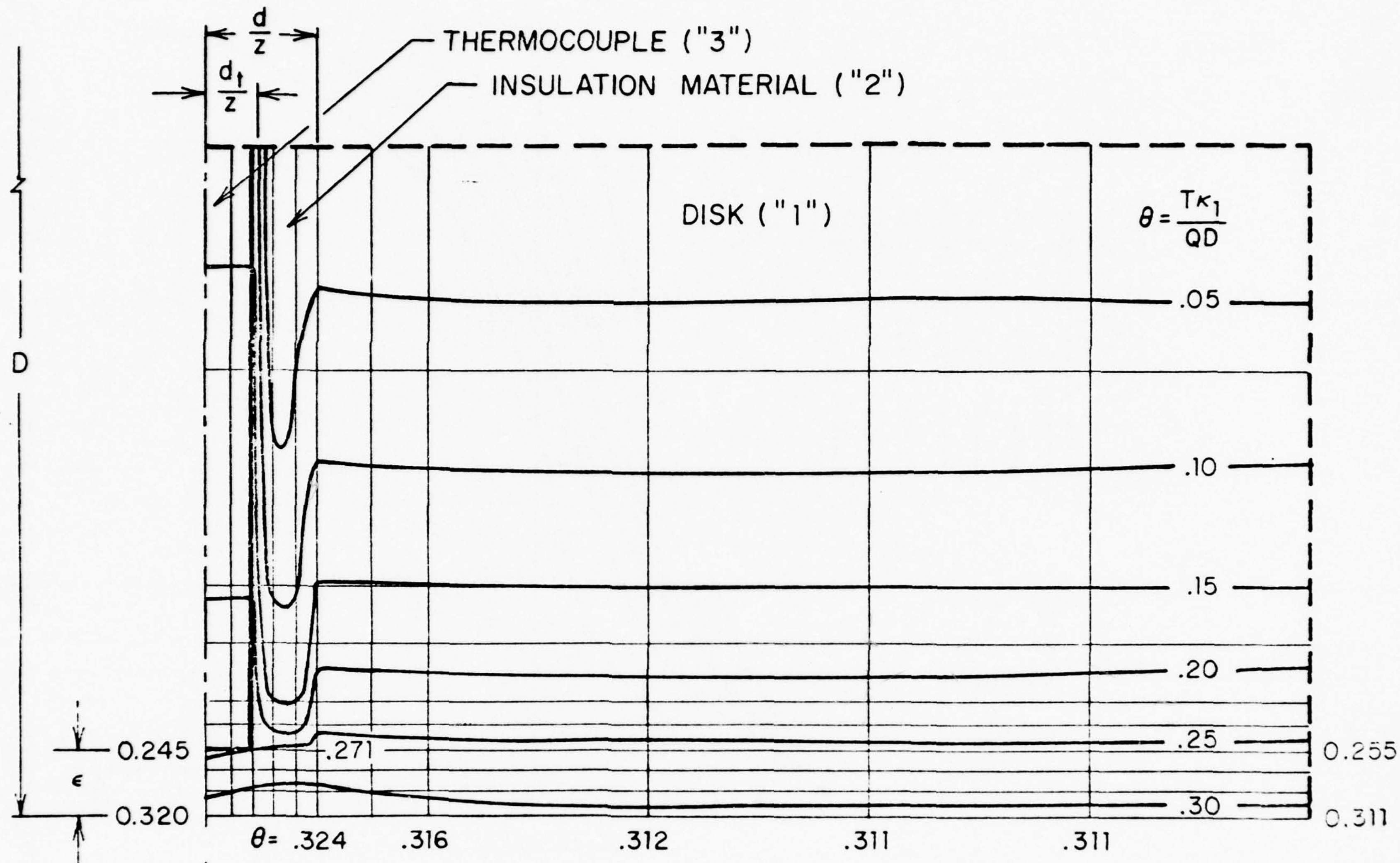
$\tau = .08,\ \frac{k_3}{\kappa_1} = 10,\ \frac{\rho c_3}{\rho c_1} = .75,\ \frac{d_t}{d} = 1.0,\ \frac{\epsilon}{D} = .06,\ \frac{d}{2D} = .1$



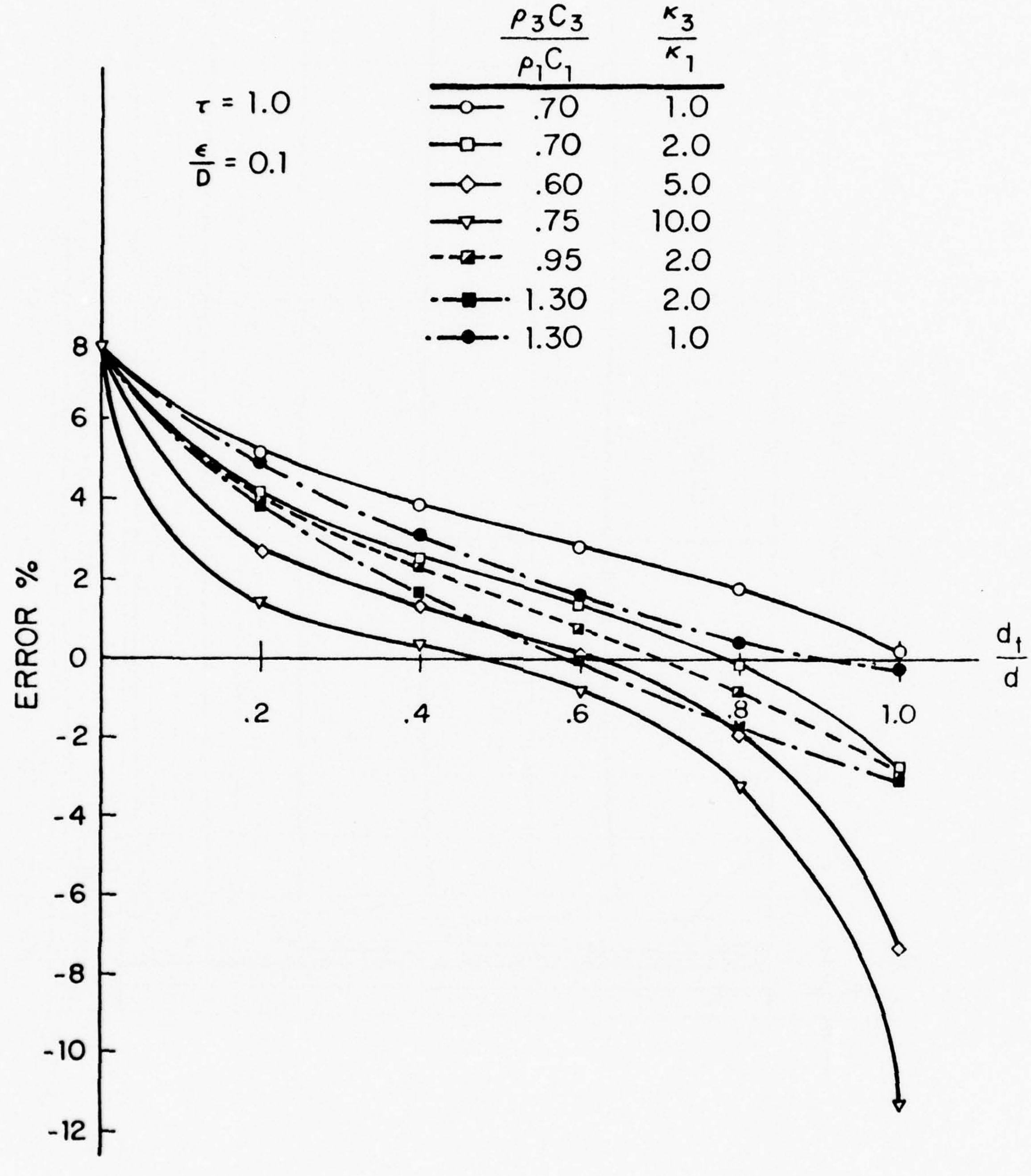
PART I Figure 5 Temperature Distribution with Thermocouple Material Filled the Cavity

$$\tau = .08,\ \frac{\kappa_2}{\kappa_1} = 10,\ \frac{\rho c_3}{\rho c_1} = .75,\ \frac{d_t}{d} = .4,\ \frac{\epsilon}{D} = .06\ \frac{d}{2D} = .1$$

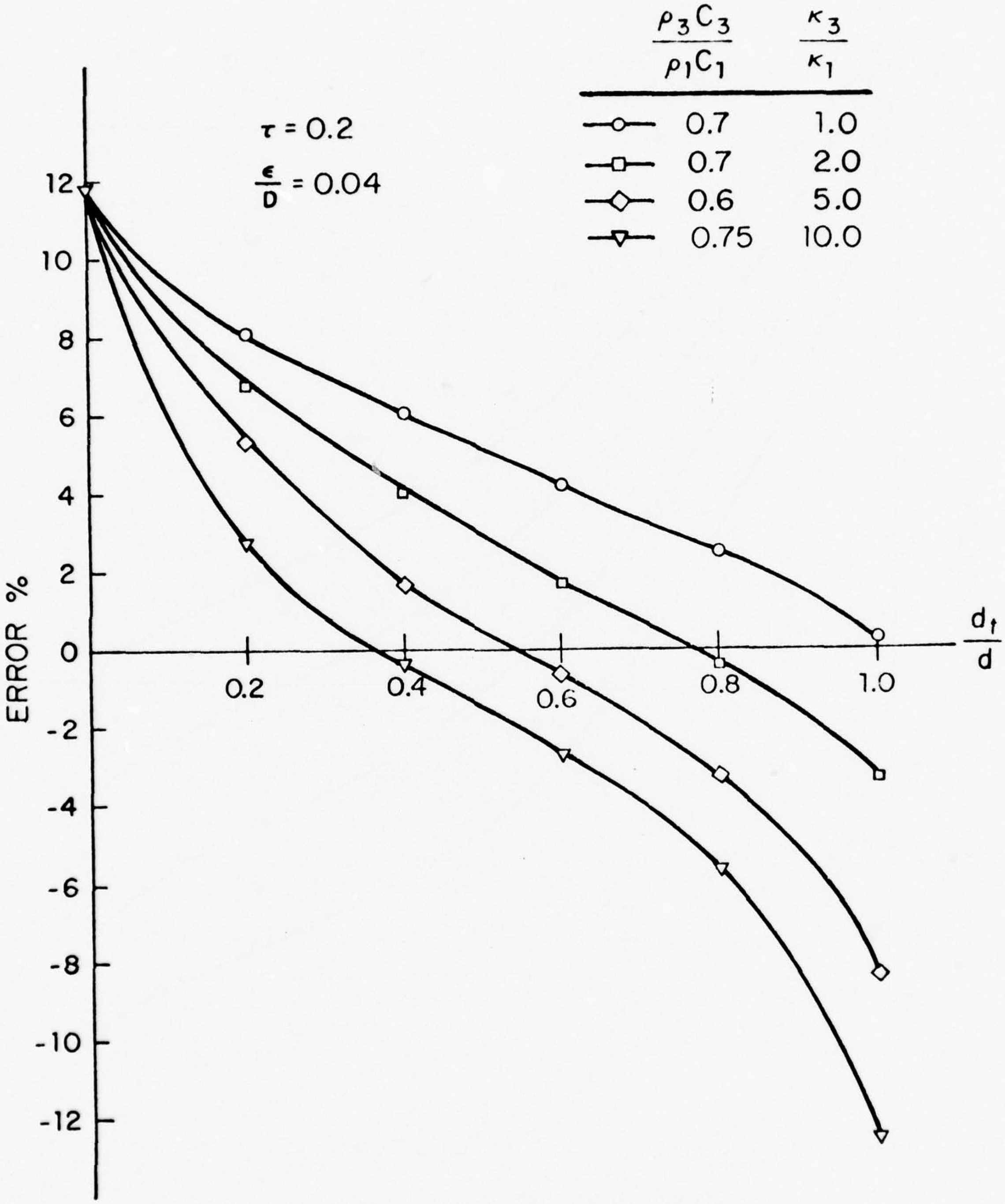
$$\frac{\kappa_2}{\kappa_1} = .005,\ \frac{\rho c_2}{\rho c_1} = .5$$



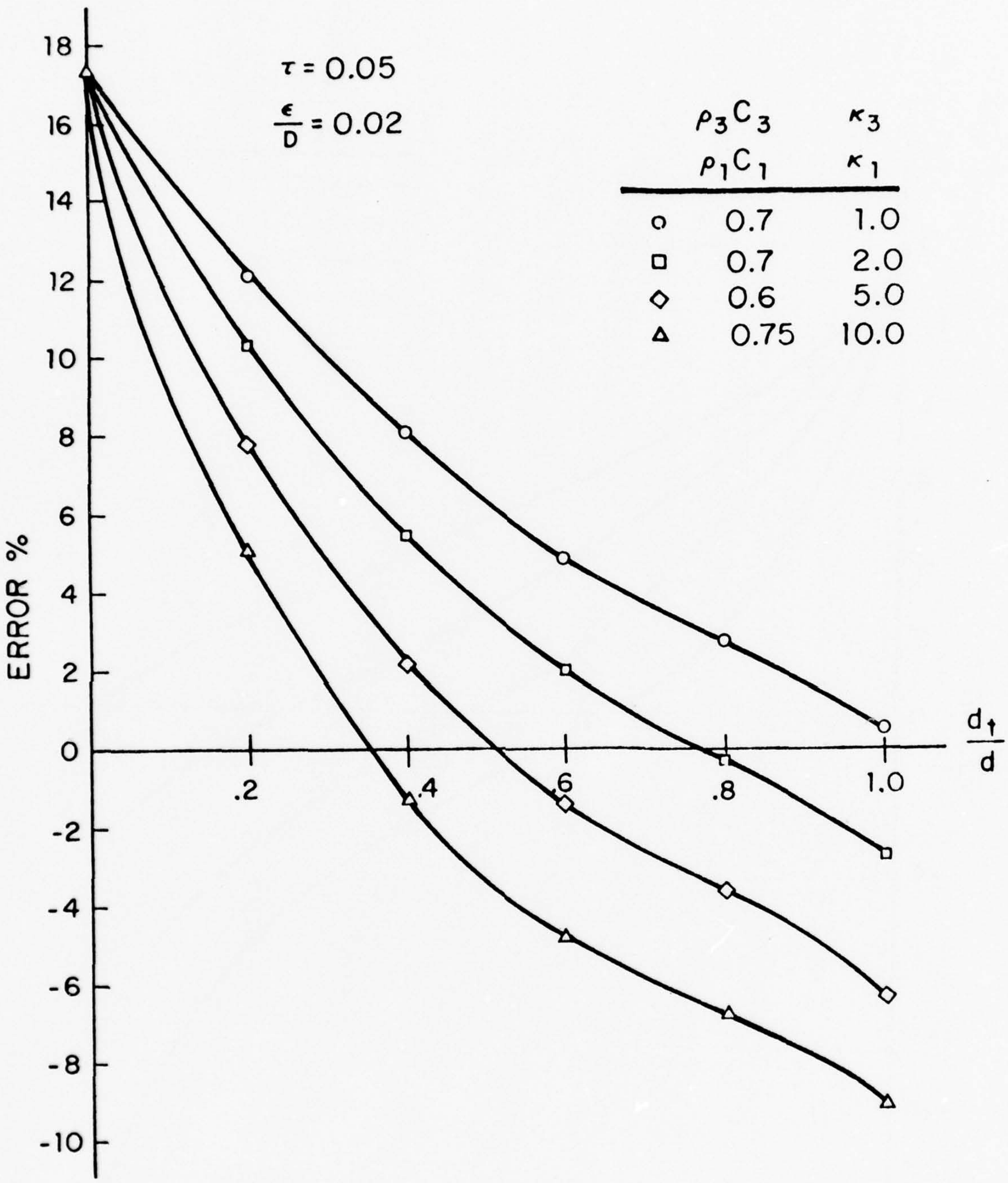
PART I Figure 6 Temperature Distribution with Thermocouple Material Partially Filled the Cavity



PART I Figure 7 Percentage Error vs. d_t/d Ratio for Various κ_3/κ_1, and ρc Ratios ϵ/D = (.1



PART I Figure 8 Percentage Error vs. d_t/d Ratio for Various κ_3/κ_1 and ρc Ratios $\varepsilon/D = 0.04$



PART I Figure 9 Percentage Error vs. d_t/d Ratio for Various κ_3/κ_1 and ρc Ratios $\varepsilon/D = 0.02$

PART I Table 1a

Commonly Used Thermocouples

Types of Thermocouples	Temperature Ranges °F	°C
Copper/Constantan	-300/750	-148/398
Iron/Constantan	-300/1600	-148/871
Chromel/Alumel	-300/2300	-148/1260
Chromel/Constantan	32/1800	0/982
Platinum 10% Rhodium/Platinum	32/2800	0/1537
Platinum 13% Rhodium/Platinum 6% Rh	100/3270	37/1798
Platinel 1813 Platinel 1503	32/2372	0/1300
Iridium/Iridium 60% Rhodium	2552/3326	1400/1830
Tungsten 3% Rhenium/Tungsten 25% Rhenium	50/4000	10/2204
Tungsten/Tungsten 26% Rhenium	60/5072	15/2800
Tungsten 5% Rhenium/Tungsten 26% Rhenium	32/5000	0/2760

PART I Table 1b

Thermal Properties of Thermocouple Materials

Metal, Insulator (subscript 3)	$K_3 \frac{cal}{sec.\ cm\ K}$	$\frac{\kappa_3}{\kappa\ steel}$	$\frac{\kappa_3}{\kappa\ aluminum}$	$\rho_3 c_3 \frac{cal}{cm^3}$	$\frac{\rho_3 c_3}{\rho c\ steel}$	$\frac{\rho_3 c_3}{\rho c\ aluminum}$
Aluminum	0.554	4.66	1.00	0.65	0.60	1.0
Copper	0.935	7.87	1.69	0.84	0.78	1.30
Chromium	0.201	1.70	0.36	0.84	0.78	1.30
Nickel	0.160	1.35	0.29	1.20	1.12	1.86
Platinum	0.174	1.46	0.31	0.74	0.69	1.14
Steel	0.119	1.0	0.21	1.08	1.00	1.66
Tungsten	0.373	3.14	0.67	0.59	0.55	0.92
Iridium	0.345	2.91	0.62	0.66	0.61	1.01
Rhodium	0.360	3.03	0.65	0.72	0.67	1.12
Rhenium	0.111	0.94	0.20	0.73	0.68	1.14
Nickel-Chromium	0.040	0.34	0.07	1.01	0.94	1.56
Copper-Nickel	0.059	0.50	0.11	0.99	0.92	1.52
Teflon	6×10^{-4}	0.005	0.001	0.54	0.5	0.83
Air	1.26×10^{-4}	0.001	0.0002	0.53	0.49	0.82

Note: The value quoted is the averaged value over 200 to 800 °K whenever data are available.

PART I Table 2

Percentage Error of Temperature at Cavity Base (/D = 0.1)

Table 2.1 κ_3/κ_1 = 0.5 ε/D = 0.1

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.9	1	1.9	1.4	1.0	0.8	0.6	0.5
	2	3.4	2.5	1.9	1.4	1.1	1.0
	4	5.1	3.8	2.9	2.2	1.7	1.5
	8	6.4	4.7	3.7	2.9	2.3	1.9
	12	6.9	5.1	4.1	3.3	2.6	2.2
	20	7.4	5.5	4.5	3.7	2.9	2.4
	40	7.8	5.9	4.8	4.0	3.2	2.6
	60	7.9	6.0	4.9	4.0	3.2	2.6
	80	7.9	6.0	4.8	3.9	3.2	2.6
	100	7.9	6.0	4.8	3.9	3.2	2.6
1.3	1		1.3	0.9	0.6	0.4	0.3
	2		2.4	1.7	1.1	0.8	0.6
	4		3.7	2.6	1.8	1.2	1.1
	8		4.6	3.4	2.4	1.6	1.5
	12		5.0	3.8	2.7	1.9	1.8
	20		5.4	4.2	3.1	2.2	2.0
	40		5.8	4.5	3.4	2.5	2.3
	60		5.9	4.5	3.4	2.5	2.3
	80		5.9	4.5	3.3	2.5	2.3
	100		5.8	4.5	3.3	2.5	2.3
1.8	1		1.2	0.7	0.4	0.1	0.1
	2		2.3	1.4	0.8	0.4	0.3
	4		3.6	2.3	1.4	0.7	0.6
	8		4.5	3.0	1.9	0.9	1.0
	12		4.9	3.4	2.1	1.1	1.2
	20		5.3	3.8	2.5	1.5	1.6
	40		5.6	4.1	2.8	1.7	1.9
	60		5.7	4.1	2.7	1.7	1.9
	80		5.7	4.1	2.6	1.6	1.9
	100		5.7	4.0	2.6	1.6	1.9

Table 2.2 $\kappa_3/\kappa_1 = 1.0$ $\varepsilon/D = 0.1$

ρ_3c_3/ρ_1c_1	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.7	1		1.2	0.8	0.5	0.3	0.2
	2		2.3	1.5	1.0	0.6	0.2
	4		3.4	2.4	1.6	1.0	0.3
	8		4.3	3.1	2.1	1.3	0.3
	12		4.6	3.3	2.3	1.5	0.3
	20		4.9	3.6	2.6	1.6	0.3
	40		5.2	3.8	2.8	1.7	0.2
	60		5.2	3.9	2.8	1.7	0.2
	80		5.2	3.9	2.8	1.8	0.2
	100		5.2	3.9	2.8	1.8	0.2
0.95	1		1.2	0.7	0.4	0.2	0.0
	2		2.2	1.4	0.8	0.3	0.0
	4		3.3	2.2	1.2	0.6	0.0
	8		4.2	2.8	1.7	0.8	0.1
	12		4.5	3.1	1.9	0.9	0.0
	20		4.8	3.4	2.2	1.1	0.0
	40		5.1	3.6	2.3	1.2	0.0
	60		5.1	2.6	2.3	1.2	0.0
	80		5.1	3.6	2.3	1.2	0.0
	100		5.1	3.6	2.3	1.2	0.0
1.3	1		1.1	0.6	0.2	-0.0	-0.1
	2		2.1	1.1	0.5	-0.0	-0.2
	4		3.2	1.8	0.8	0.0	-0.3
	8		4.0	2.4	1.2	0.2	-0.3
	12		4.4	2.7	1.4	0.3	-0.3
	20		4.7	3.0	1.6	0.4	-0.3
	40		5.0	3.2	1.7	0.4	-0.2
	60		5.0	3.2	1.7	0.4	-0.2
	80		5.0	3.2	1.6	0.4	-0.2
	100		5.0	3.1	1.6	0.4	-0.2

Table 2.3 $\kappa_3/\kappa_1 = 5.0$ $\varepsilon/D = 0.1$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.6	1		0.7	0.1	-0.3	-0.5	-1.0
	2		1.3	0.1	-0.4	-0.9	-1.8
	4		2.1	0.5	-0.6	-1.4	-3.0
	8		2.5	0.6	-0.7	-1.9	-4.5
	12		2.5	0.7	-0.7	-2.0	-5.4
	20		2.6	0.9	-0.4	-2.0	-6.4
	40		2.6	1.2	-0.1	-1.9	-7.2
	60		2.7	1.4	-0.2	-1.8	-7.3
	80		2.7	1.4	-0.2	-1.8	-7.3
	100		2.7	1.4	-0.2	-1.8	-7.3

Table 2.4 $\kappa_3/\kappa_1 = 10$ $\varepsilon/D = 0.1$

0.75	1	0.3	-0.4	-0.8	-1.0	-1.4
	2	0.7	-0.6	-1.4	-1.9	-2.7
	4	1.1	-0.8	-2.0	-2.9	-4.6
	8	1.3	-0.9	-2.4	-3.7	-6.8
	12	1.3	-0.8	-2.3	-3.8	-8.3
	20	1.2	-0.4	-1.8	-3.7	-9.9
	40	1.4	0.2	-1.0	-3.3	-11.1
	60	1.4	0.3	-0.9	-3.2	-11.3
	80	1.4	0.4	-0.8	-3.2	-11.3
	100	1.4	0.4	-0.8	-3.2	-11.3

PART I Table 3

Percentage Error of Temperature at Cavity (/D = 0.04)

Table 3.1 $\kappa_3/\kappa_1 = 0.5$ $\varepsilon/D = 0.04$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.9	.2	1.5	0.9	0.7	0.5	0.4	0.4
	.4	3.2	2.0	1.5	1.1	0.8	0.7
	.8	5.6	3.9	2.8	2.0	1.5	1.3
	1.6	8.2	6.0	4.5	3.3	2.4	1.9
	2.4	9.3	7.0	5.3	3.9	2.9	2.3
	4.0	10.3	7.8	6.1	4.5	3.4	2.7
	8.0	11.2	8.5	6.7	5.2	3.9	3.1
	12.0	11.5	8.8	7.1	5.5	4.2	3.3
	16.0	11.8	9.0	7.3	5.8	4.5	3.4
	20.0	11.9	9.1	7.5	6.0	4.6	3.5
1.3	.2		0.8	0.5	0.3	0.2	0.2
	.4		1.9	1.2	0.8	0.5	0.4
	.8		3.7	2.5	1.6	1.0	0.8
	1.6		5.8	4.0	2.6	1.6	1.3
	2.4		6.8	4.9	3.2	2.1	1.6
	4.0		7.6	5.6	3.8	2.5	2.0
	8.0		8.4	6.3	4.4	2.9	2.5
	12.0		8.6	6.6	4.7	3.2	2.7
	16.0		8.8	6.9	5.0	3.4	2.9
	20.0		9.0	7.0	5.2	3.6	3.0
1.8	.2		0.7	0.4	0.2	0.1	0.0
	.4		1.7	0.9	0.5	0.2	0.1
	.8		3.4	2.1	1.1	0.5	0.3
	1.6		5.5	3.5	2.0	0.9	0.6
	2.4		6.5	4.3	2.5	1.2	0.9
	4.0		7.4	5.1	3.1	1.6	1.2
	8.0		8.2	5.8	3.6	1.9	1.7
	12.0		8.5	6.1	3.9	2.1	2.0
	16.0		8.7	6.3	4.1	2.3	2.2
	20.0		8.8	6.5	4.3	2.5	2.4

Table 3.2 κ_3/κ_1 = 1.0 ε/D = 0.04

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.7	0.2		0.8	0.5	0.3	0.2	0.1
	0.4		1.8	1.1	0.7	0.4	0.3
	0.8		3.4	2.2	1.4	0.8	0.4
	1.6		5.4	3.6	2.2	1.3	0.4
	2.4		6.3	4.3	2.7	1.6	0.4
	4.0		7.1	5.0	3.2	1.9	0.4
	8.0		7.6	5.5	3.7	2.2	0.4
	12.0		7.9	5.8	4.0	2.4	0.4
	16.0		8.0	5.9	4.1	2.4	0.4
	20.0		8.1	6.1	4.2	2.5	0.3
0.95	0.2		0.7	0.4	0.2	0.1	0.0
	0.4		1.6	0.9	0.5	0.2	0.0
	0.8		3.3	1.9	1.0	0.4	0.1
	1.6		5.2	3.3	1.8	0.7	0.1
	2.4		6.1	3.9	2.2	0.9	0.1
	4.0		6.9	4.6	2.6	1.1	0.1
	8.0		7.5	5.1	3.1	1.4	0.1
	12.0		7.7	5.4	3.3	1.6	0.1
	16.0		7.9	5.6	3.5	1.6	0.1
	20.0		8.0	5.7	3.6	1.7	0.1
1.3	0.2		0.6	0.3	0.0	-0.1	-0.1
	0.4		1.5	0.7	0.2	-0.1	-0.2
	0.8		3.1	1.6	0.6	-0.1	-0.3
	1.6		5.0	2.8	1.2	0.1	-0.4
	2.4		5.9	3.5	1.5	0.2	-0.4
	4.0		6.7	4.1	1.0	0.3	-0.4
	8.0		7.3	4.6	2.2	0.4	-0.4
	12.0		7.6	4.9	2.5	0.5	-0.4
	16.0		7.7	5.1	2.7	0.6	-0.4
	20.0		7.9	5.2	2.8	0.7	-0.3

Table 3.3 $\kappa_3/\kappa_1 = 5$ $\varepsilon/D = 0.04$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	.10
	$\tau \times 10^2$			Error %			
0.6	0.2		0.3	-0.1	-0.3	-0.4	-0.6
	0.4		0.9	0.0	-0.5	-0.8	-1.1
	0.8		2.0	0.4	-0.6	-1.2	-2.1
	1.6		3.4	0.9	-0.7	-1.8	-3.2
	2.4		4.1	1.2	-0.8	-2.1	-4.0
	4.0		4.6	1.4	-0.9	-2.5	-5.0
	8.0		4.7	1.4	-1.1	-3.0	-6.4
	12.0		4.7	1.4	-1.0	-3.2	-7.3
	16.0		4.7	1.6	-0.8	-3.2	-7.9
	20.0		4.7	1.7	-0.6	-3.2	-8.3

Table 3.4 $\kappa_3/\kappa_1 = 10$ $\varepsilon/D = 0.04$

0.75	0.2		0.0	-0.4	-0.6	-0.7	-0.9
	0.4		0.3	-0.6	-1.1	-1.4	-1.7
	0.8		1.0	-0.7	-1.7	-2.3	-3.0
	1.6		2.1	-0.8	-2.5	-3.5	-4.6
	2.4		2.6	-0.8	-2.9	-4.2	-5.7
	4.0		2.9	-0.9	-3.4	-5.0	-7.2
	8.0		2.9	-1.0	-3.7	-5.8	-9.5
	12.0		2.8	-0.8	-3.4	-5.8	-11.0
	16.0		2.8	-0.5	-3.1	-5.7	-11.9
	20.0		2.8	-0.3	-2.7	-5.6	-12.5

PART I Table 4

Percentage Error of Temperature at Cavity Base ($\epsilon/D = 0.02$)

Table 4.1 $\kappa_3/\kappa_1 = 0.5$ $\epsilon/D = 0.02$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
1.3	0.05		0.3	0.1	0.1	0.1	0.0
	0.10		0.8	0.4	0.2	0.2	0.1
	0.20		2.0	1.2	0.7	0.5	0.4
	0.40		4.5	2.7	1.6	1.0	0.8
	0.60		6.5	4.0	2.3	0.4	1.1
	1.00		8.9	5.6	3.3	2.0	1.5
	2.00		11.4	7.6	4.7	2.8	2.2
	3.00		12.4	8.5	5.4	3.3	2.6
	4.00		12.9	9.0	5.8	3.6	2.8
	5.00		13.1	9.3	6.0	3.7	3.0

Table 4.2 $\kappa_3/\kappa_1 = 1$ $\varepsilon/D = 0.02$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.7	0.05		0.3	0.2	0.1	0.1	0.1
	0.10		0.8	0.5	0.3	0.2	0.2
	0.20		1.9	1.1	0.7	0.4	0.3
	0.40		4.3	2.5	1.5	0.8	0.5
	0.60		6.1	3.6	2.1	1.1	0.5
	1.00		8.3	5.1	2.9	1.5	0.6
	2.00		10.6	6.8	3.9	2.1	0.6
	3.00		11.4	7.5	4.4	2.4	0.5
	4.00		11.8	7.8	4.7	2.5	0.5
	5.00		12.1	8.1	4.9	2.8	0.5
0.95	0.05		0.2	0.1	0.0	0.0	0.0
	0.10		0.7	0.3	0.1	0.1	0.0
	0.20		1.8	0.9	0.4	0.1	0.0
	0.40		4.0	2.1	1.0	0.3	0.1
	0.60		5.8	3.1	1.5	0.5	0.1
	1.00		8.0	4.5	2.2	0.8	0.1
	2.00		10.3	6.1	3.1	1.2	0.1
	3.00		11.2	6.8	3.5	1.4	0.1
	4.00		11.6	7.2	3.8	1.5	0.1
	5.00		11.8	7.4	4.0	1.7	0.1
1.3	0.05		0.2	0.0	-0.0	-0.1	-0.1
	0.10		0.5	0.1	-0.1	-0.1	-0.1
	0.20		1.6	0.6	0.1	-0.2	-0.3
	0.40		3.7	1.6	0.5	-0.2	-0.4
	0.60		5.4	2.5	0.8	-0.1	-0.5
	1.00		7.6	3.8	1.4	-0.0	-0.5
	2.00		10.0	5.4	2.1	0.2	-0.5
	3.00		10.9	6.0	2.5	0.3	-0.5
	4.00		11.3	6.4	2.7	0.4	-0.5
	5.00		11.5	6.7	2.9	0.4	-0.5

Table 4.3 $\kappa_3/\kappa_1 = 10$ $\varepsilon/D = 0.02$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.75	0.05	0.7	0.0	-0.26	-0.33	-0.37	-0.41
	0.10	1.7	0.0	-0.48	-0.66	-0.76	-0.86
	0.20	4.9	0.2	-0.72	-0.93	-1.40	-1.61
	0.40	9.0	1.2	-0.85	-1.78	-2.28	-2.71
	0.60	11.6	2.0	-0.86	-2.20	-2.89	-2.50
	1.00	13.2	3.2	-0.84	-2.78	-3.76	-4.66
	2.00	15.8	4.5	-0.88	-3.66	-5.06	-6.44
	3.00	16.7	4.9	-0.99	-4.20	-5.84	-7.56
	4.00	17.1	5.1	-1.10	-4.57	-6.38	-8.42
	5.00	17.4	5.1	-1.20	-4.80	-6.75	-9.12

Table 4.4 $\kappa_3/\kappa_1 = 5.0$ $\varepsilon/D = 0.02$

0.6	0.05	0.7	0.05	-0.00	-0.17	-0.20	-0.25
	0.10	1.7	0.24	-0.15	-0.32	-0.43	-0.55
	0.20	4.9	0.87	-0.06	-0.53	-0.80	-1.09
	0.40	9.0	1.33	0.34	-0.70	-1.29	-1.89
	0.60	11.6	3.53	0.71	-0.78	-1.61	-2.48
	1.00	13.2	5.12	1.25	-0.88	-2.04	-3.33
	2.00	15.8	6.87	1.92	-1.00	-2.62	-4.59
	3.00	16.7	7.48	2.15	-1.12	-2.97	-5.33
	4.00	17.1	7.72	2.21	-1.25	-3.26	-5.88
	5.00	17.4	7.80	2.20	-1.36	-3.49	-6.32

PART II

IMPROVED ACCURACY IN THE PREDICTION OF SURFACE HEAT FLUX AND TEMPERATURE BY AN INTRINSIC THERMOCOUPLE

LIST OF FIGURES - PART II

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INTRODUCTION

In the study of transient heat transfer, many experimental difficulties may arise if heat flux sensors or thermocouples are installed direct at the surface of a body. For example, a probe may be damaged by a piston or a projectile sliding over a cylinder or barrel. A probe on a melting and ablative surface of heat shield can be easily destroyed because of high temperature. Furthermore a surface probe exposed to both radiative and convective environment may measure an erroneous surface heat flux and temperature if the probe has a different radiative property from that of the measured surface. In these circumstances, calculation of the transient surface heat flux and the surface temperature can be achieved by inverting a temperature history measured at some location inside the body.

In general, the prediction of a surface heat flux and temperature by the measured data at some location interior to a body is known as the "inverse problem". Many configurations, such as spheres, cylinders, and slabs, had been studied by many workers and many methods such as numerical, graphical, series, convolution integral, and Laplace transforms were used. Stolz [1], Beck [2] and Williams and Curry [3], considered the numerical inversion of the integral solution for semi-infinite and other bodies. In this method, care is required in selecting a time interval in order to achieve a stable solution. Carslaw and Jaeger [4], Burggraf [5], Koveryanov [6], and Shumakov [7], respectively considered different series approaches in which generally the local heat flux at an interior location and their higher derivaties are required. However, it is difficult to measure experimentally or to process the measured data for the derivative

of the temperature. Sparrow, Haji-Sheikh, and Lundgren [8], Imber and Kahn [9], Imber [10], Sabherwal [11], Masket and Vastano [12], Deverall and Channapragada [13] and Chen and Thomsen [14] applied the transform method. In these works, the solution is represented in either an integral form after some manipulation of the contour integral from the inverse transform, or in a series form after an expansion of the solution for small and large times. Using Laplace transformation Chen and Thomsen [14] introduced a polynomial in terms of an error function to represent the response of thermocouple measurement and the inversion is accomplished for any transient surface heat flux at the inner surface of a cylindrical tube. In their study, the cylindrical thickness was assumed to be relatively thick such that the temperature at a large distance from the heating surface remains constant. Therefore, only one interior temperature response near the surface was needed in the experimental measurement. Their inversion solution however was valid only for a short duration due to the asymptotic expansion of the modified Bessel function in the inverse Laplace transform. Chen and Chiou [15] studied the inversion problem for the case of a semi-infinite slab or a thick slab using a Laplace transformation. The exact solution was obtained from the inverse Laplace transform for any time interval. It was then shown that their analysis may be approximately applied to the case of the hollow cylinder if the interior temperature response is measured at a location close to the inner wall.

This report presents (a) the improved numerical solution of the inversion solution reported by Chen and Chiou [15] and (b) a further demonstration of the capability of the solution.

The theoretical analysis of Chen and Chiou [15] is recapitulated in Appendix A in which the surface heat flux and temperature is predicted by inverting a temperature history measured at some location inside the solid body. The inversion solution is obtained by invoking Laplace transformation. Both the surface heat flux and temperature are given by Eqs. (19) and (20) in Appendix A.

It was thought that the accuracy of the computer program generated for the solution in the previous report by Chen and Chiou [15] can be improved further for the following reasons. First, the coefficients b_n (see Appendix A Eq.(11)) in the previous formulation has a dimension of temperature. Therefore the determination of the coefficients depends on the temperature range of each particular experiment. It was found that the absolute value of the coefficients b_n in some cases can become as large as an order of 10^{44}. Therefore during the subsequently numerical manipulation in the computer program error due to round off and the standard fixed up when an underflow occurred may become appreciable. To remedy this difficulty the dimensionless formulation is introduced in the analysis (Appendix A) in which the coefficient b_n is also made dimensionless. As a result the magnitude of the coefficient b_n can be greatly reduced. Secondly, the double precision format was not used throughout the previous computer program. It is felt that further accurate results may be obtained if the double precision format is adopted in the program.

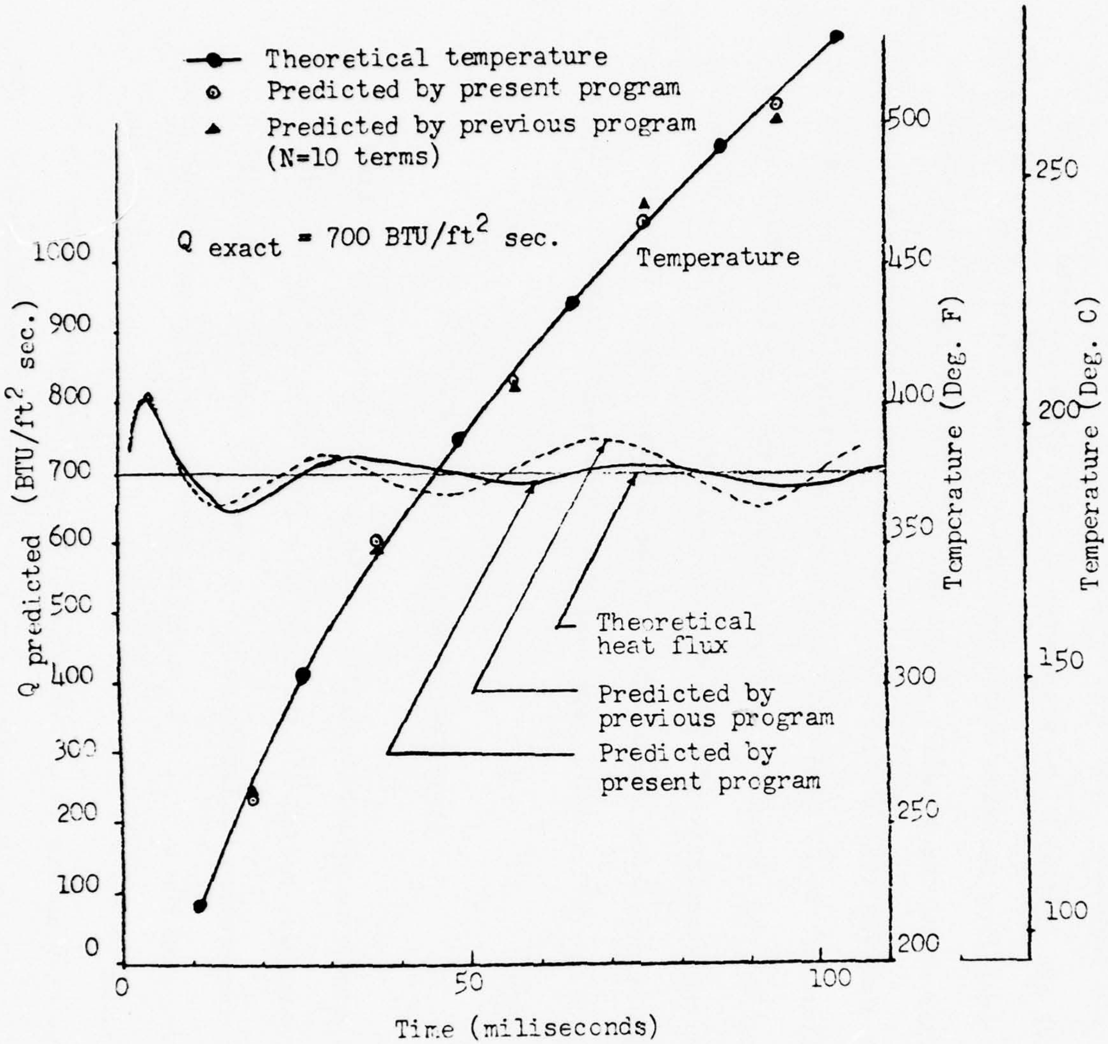
In the following section the new computer solution is shown to be indeed more accurate. Later the solution is shown to be capable of predicting a case involving a periodic surface heat flux or periodic temperature variation.

RESULTS OF THE IMPROVED COMPUTER PROGRAM

The previous computer program of Chen and Chiou [15] was recast in dimensionless form and written in the double precision format. The new computer program is listed in Appendix B. The results predicted by the new and previous computer program are given in Appendix C and shown in Figure 1 for the case of the constant surface heat flux. This is the case in which a steel slab initially at a uniform temperature is suddenly subjected to a constant heat flux Q at one of the surfaces and kept at the initial temperature on the other surface. Figure 1 shows the solution predicted by inverting the temperature response at an interior of the slab from the new and previous computer program. This solution predicted by the new and previous programs used the ten term representation for the thermocouple response. The comparison clearly shows the improvement of the new solution over the previous one. Except for the short time duration the solution with the new program reduces the error to only one half of the error of the previous program i.e., an error of less than one percent. In the short time period the solution exhibits a Gibbs phenomenon* because of the discontinuity of the surface temperature gradient occurred at initial condition. The solution shows a 17% of initial overshot of heat flux and then a 7.8% of undershot before the solution approaches the constant heat flux. It should be remarked that Gibbs phenomenon is artificially

---

* Gibbs phenomena [3]: for a sequence of transformation $T_n(t)$, $n = 1, 2, \ldots$ of a function $q(t)$ (here $q(t)$ = constant) if the interval $\lim_{t \to t_o} \inf_{n \to \infty} T_n(t)$, $\lim_{t \to t_o} \sup_{n \to \infty} T_n(t)]$ contains points outside the interval $[\lim_{t \to t_o} \inf q(t)$, $\lim_{t \to t_o} \sup q(t)]$ then the sequence is said to exhibit a Gibbs phenomena.



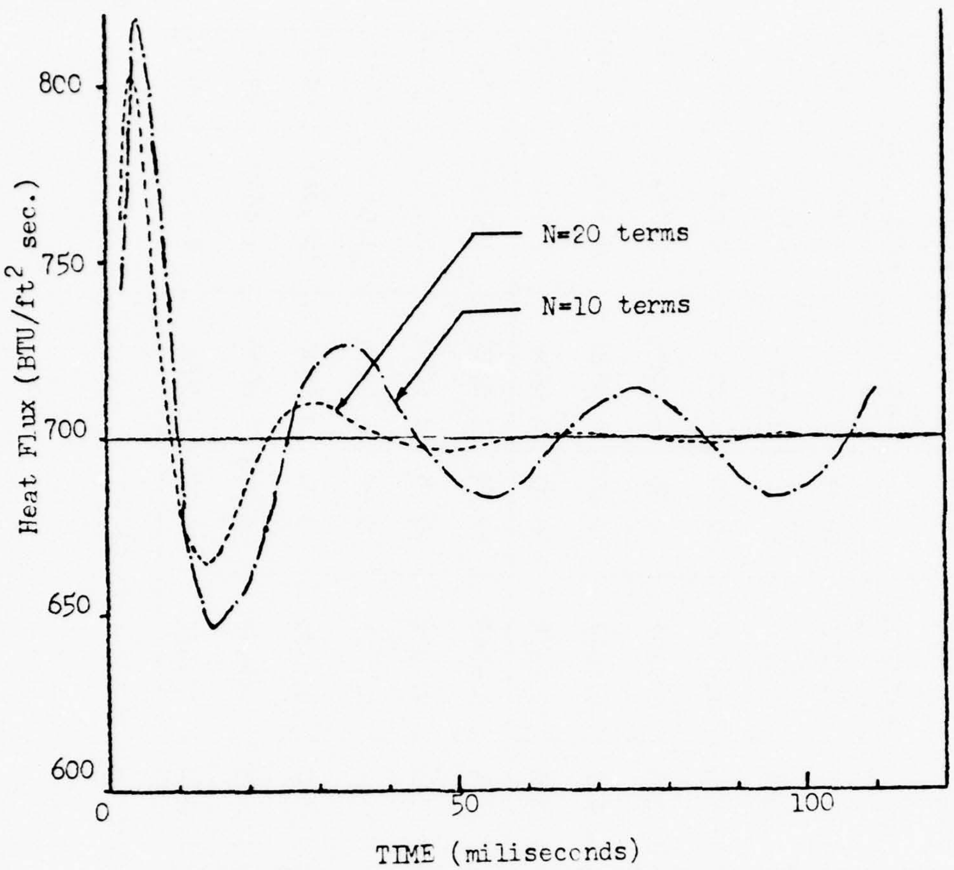
PART II Figure 1 (Comparison of Previous and Present Programs)

introduced due to the idealization of the initial condition. In most practical situations the surface heat flux will be continuous. Therefore, Gibbs phenomenon will not appear.

Figure 2 (see Table 1 also) shows the comparison between the solutions for the constant heat flux case with 10 and 20 term representation for the thermocouple response. One sees that the solution with 20 term representation after the initial Gibbs phenomenon quickly approaches the expected constant heat flux solution with a negligible error of less than 0.14 percent. This shows the accuracy of the new computer program. From Figure 2 one also observes that both overshoot and undershoot of Gibbs phenomenon are smaller for the 20 term representation. Additionally the points of the overshoot and the undershoot have moved to near the zero time which agrees with the characteristic of Gibbs phenomenon. According to Gibbs phenomenon the point of overshoot should approach the initial zero if the number of terms of the series which represent the thermocouple response is increased to infinite.

VERIFICATION OF OSCILLATORY SOLUTION

As a measure of applicability of the present inversion solution, a test problem was solved for the case of a slab subjected to a periodic surface temperature variation on one surface and held to the initial temperature on the other. The analytic solution for the problem is given in Appendix IV where a more suitable form of the solution than the one given by Carslaw and Jaeger [4] is derived and tabulated for the thermocouple response at one tenth of the slab thickness from the surface. The



PART II Figure 2 Comparison of Different Polynomial Representations

PART II TABLE 1

Comparision of Inversion Prediction and Exact Solution

N	t	f(t) $\theta(1,t)$	$\theta(0,t)$ EXACT	$\theta(0,t)$ PREDICTED	ERROR* %	$\frac{\partial\theta}{\partial x}(0,t)$ EXACT	$\frac{\partial\theta}{\partial x}(0,t)$ PREDICTED	ERROR* %
20	0.1	0.0624	0.3758	0.3773	+0.399	1.0000	0.9873	-1.27
	0.2	0.1628	0.5315	0.5314	+0.019	1.0000	1.0014	+0.14
	0.4	0.3395	0.7516	0.7516	0.00	1.0000	1.0001	+0.01
	0.6	0.4882	0.9205	0.9205	0.00	1.0000	1.0000	0.00
	0.8	0.6183	1.0629	1.0628	-0.009	1.0000	0.9996	-0.04
	1.0	0.7353	1.1884	1.1878	-0.05	1.0000	1.9986	-0.14
10	0.1	0.0624	0.3758	0.4048	+7.71	1.0000	1.0796	+7.96
	0.2	0.1628	0.5315	0.5284	-0.58	1.0000	0.9391	-6.09
	0.4	0.3395	0.7516	0.7516	0.00	1.0000	1.0179	+1.79
	0.6	0.4882	0.9205	0.9205	0.00	1.0000	0.9923	-0.77
	0.8	0.6183	1.0629	1.0626	-0.028	1.0000	1.0043	+0.43
	1.0	0.7353	1.1884	1.1887	+0.025	1.0000	0.9963	-0.37

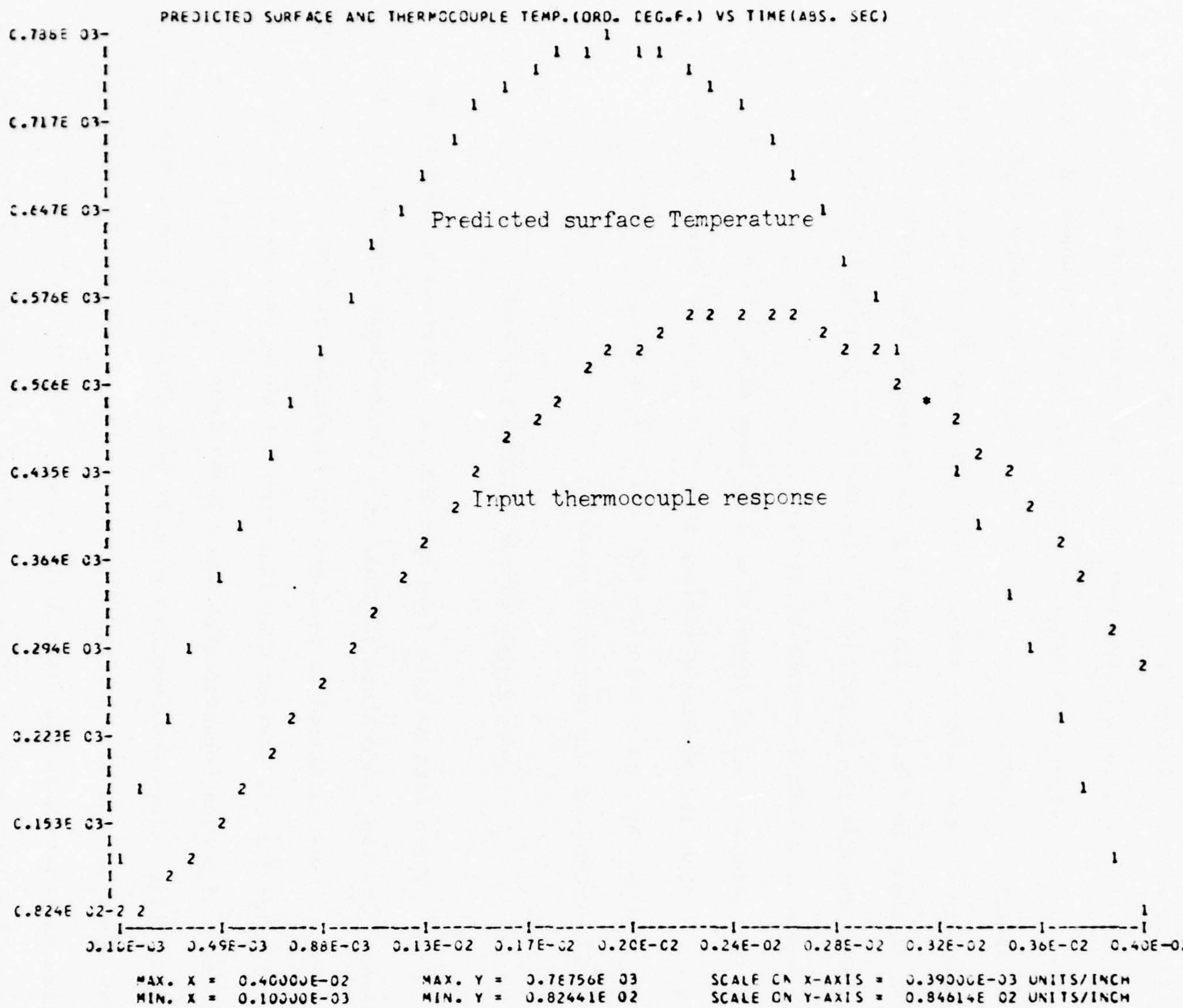
*ERROR % = ((PREDICTED)-(EXACT))/(EXACT)

surface is subjected to a periodic temperature variation with a period of 8 milliseconds. Fifteen data points of the temperature response at the thermocouple location are then input to the inversion program for prediction of the surface temperature and heat flux. The result is shown in Figure 3 and Appendix C where the data symbol "2" denotes the thermocouple response and "1" the surface temperature. The accuracy of the inversion program is shown in Table 2. Except for the extremely short time period of 0.4 milliseconds the prediction by the inversion program with 15 term representation is within 2 percent of error.

The accuracy can be improved more if more data points are used. Figure 4 shows the predicted surface heat flux which we were unable to compute from the series solution (Eq. (7) of Appendix D). This demonstrates the versatility of the inversion solution.

APPLICATION OF THE INVERSION PROGRAM

Three sets of data (see Appendix C) provided by Rock Island Arsenal for the temperature response of a thermocouple embedded in a M60 gun barrel were utilized to evaluate the inversion solution. The inversion prediction for the surface heat flux from all three sets of data were extremely high when compared with other known data calculated by Chen and Chiou [15]. Since the program correctly predicted the surface heat flux for other sets of experimental data it was judged that the three sets of data may contain inaccurate initial time. For most experimentations the recording instrument is likely to experience some delay in responding to the extremely fast transient heat flux typical in gun bores. Therefore an advanced shift of time of 2 milliseconds in the data was tested. The



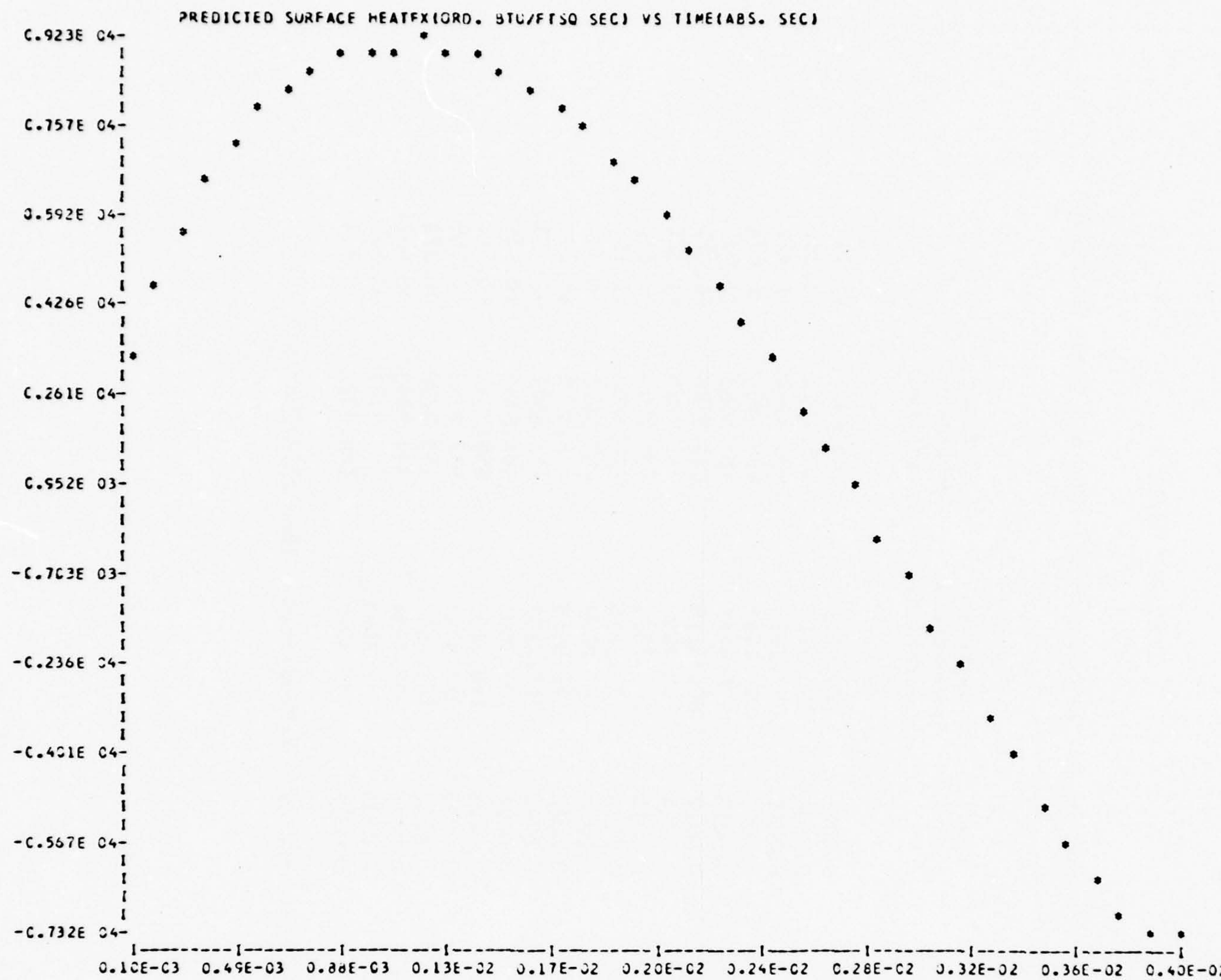
PART II Figure 3 Predicted Surface Temperature

PART II Table 2

Comparison of Inversion Prediction and Exact Solution

Time (sec.)	Surface Temperature (theoretical)	Surface Temperature (predicted)	Error* %
0.0002	189.5041	193.4060	+3.563
0.0004	296.3119	301.4311	+2.367
0.0006	397.7934	403.8998	+1.921
0.0008	491.4497	498.3988	+1.689
0.0010	574.9747	582.5683	+1.534
0.0012	646.3119	654.3358	+1.417
0.0014	703.7046	711.9396	+1.321
0.0016	745.7396	753.9650	+1.236
0.0018	771.3818	779.3802	+1.157
0.0020	780.0000	787.5615	+1.08
0.0022	771.3818	778.3093	+1.00
0.0024	745.7396	751.8527	+0.918
0.0026	703.7046	408.8443	+0.874
0.0028	646.3119	650.3441	+0.712
0.0030	574.9747	577.7933	+0.569
0.0032	491.6697	492.9790	+0.372
0.0034	397.7934	397.9893	+0.0616
0.0036	296.3119	295.2761	-0.679
0.0038	189.5041	188.8522	-0.595

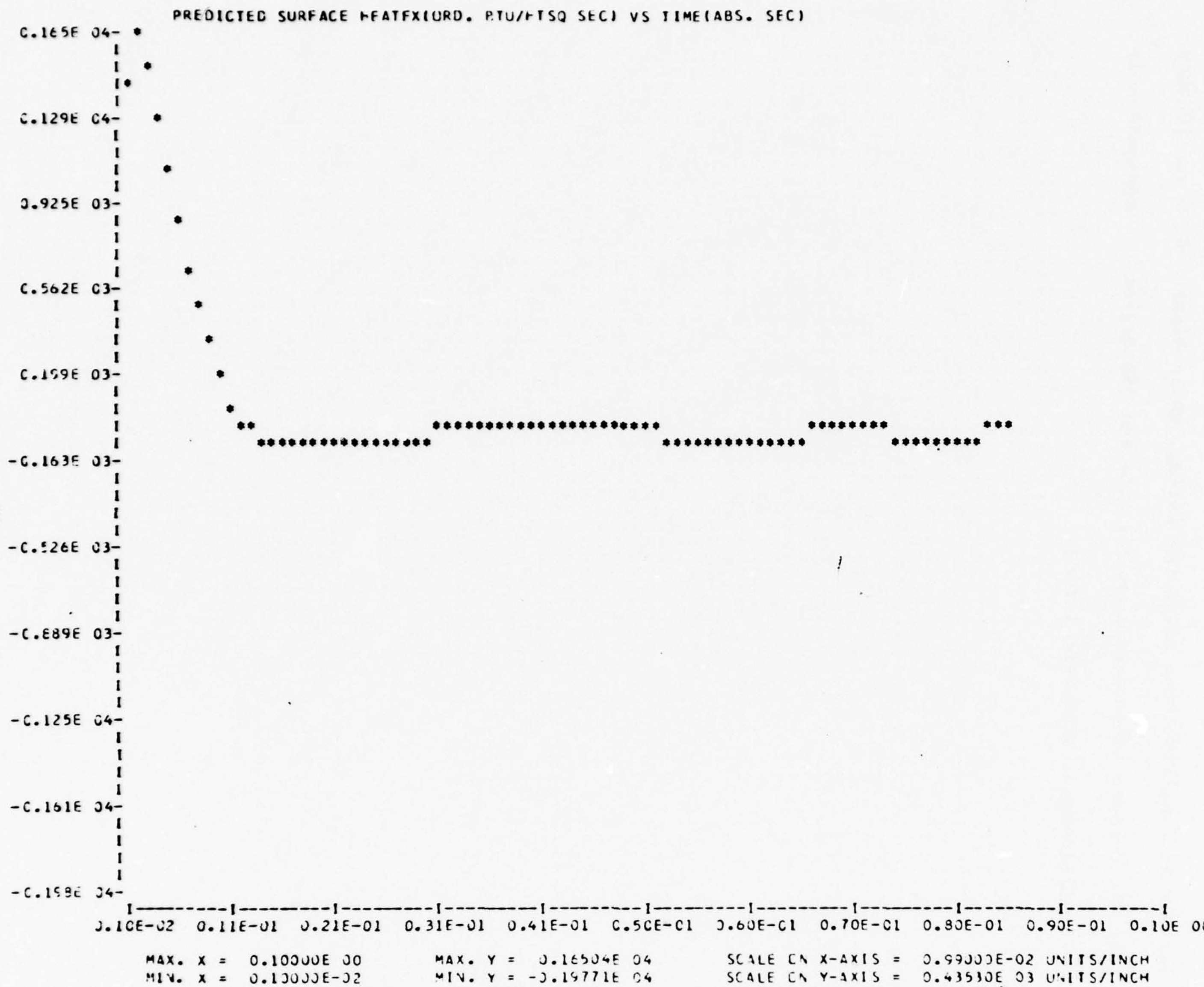
*Error = ((Predicted - (Exact))/(Exact)) x 100



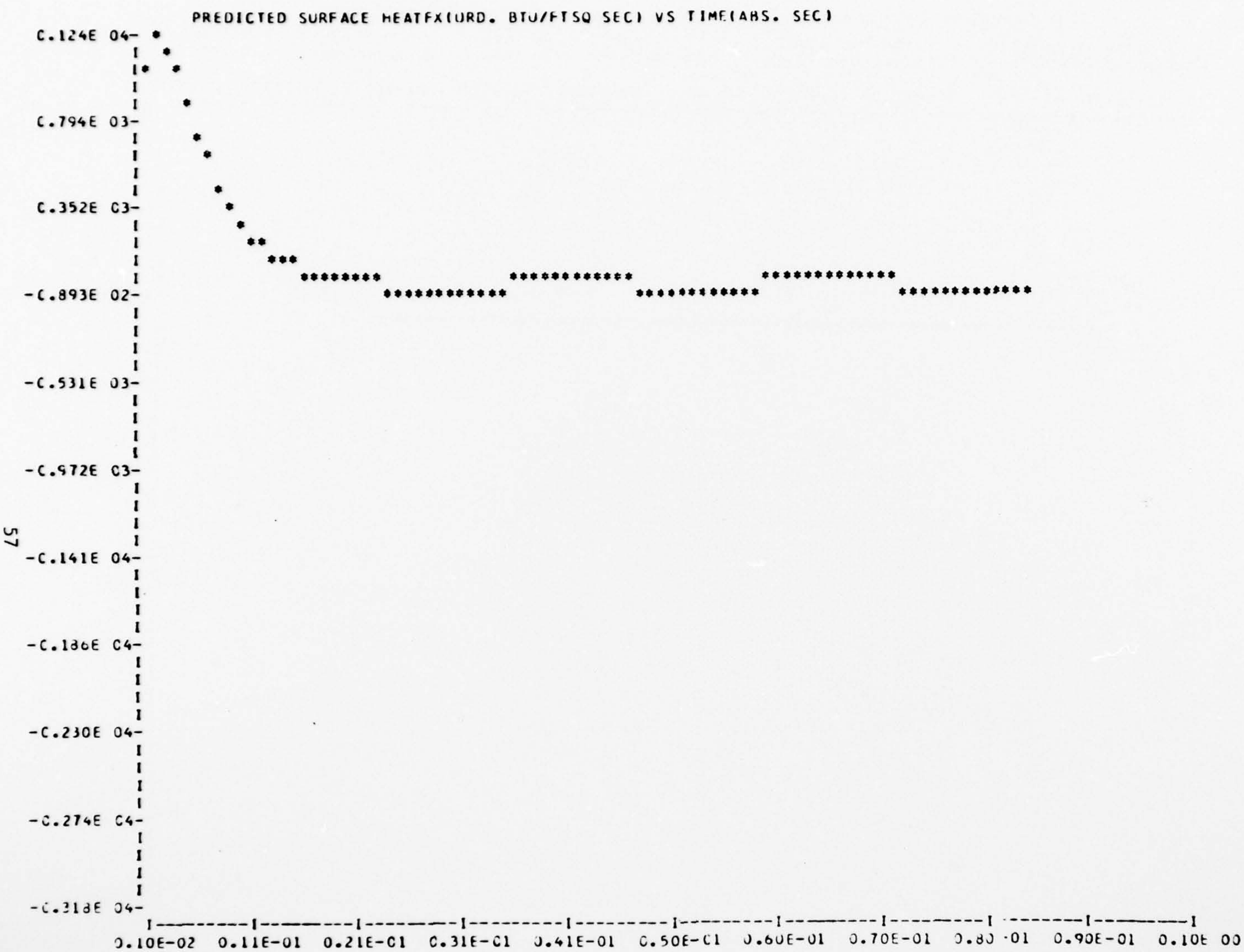
MAX. X = 0.40000E-02 MAX. Y = 0.92277E 04 SCALE ON X-AXIS = 0.39000E-03 UNITS/INCH
MIN. X = 0.10000E-03 MIN. Y = -0.73231E 04 SCALE ON Y-AXIS = 0.19861E 04 UNITS/INCH

PART II Figure 4 Predicted Surface Heat Flux

result is shown in Appendix C-3. Figures 5, 6 and 7 respectively show the predicted maximum heat flux of 1650, 1235 and 2695 Btu/ft^2 sec. approximately at 2 milliseconds after the firing, while Figures 8, 9 and 10 show that the surface temperatures are 273, 234 and 396 degree F. approximately at 6 milliseconds after the firing.



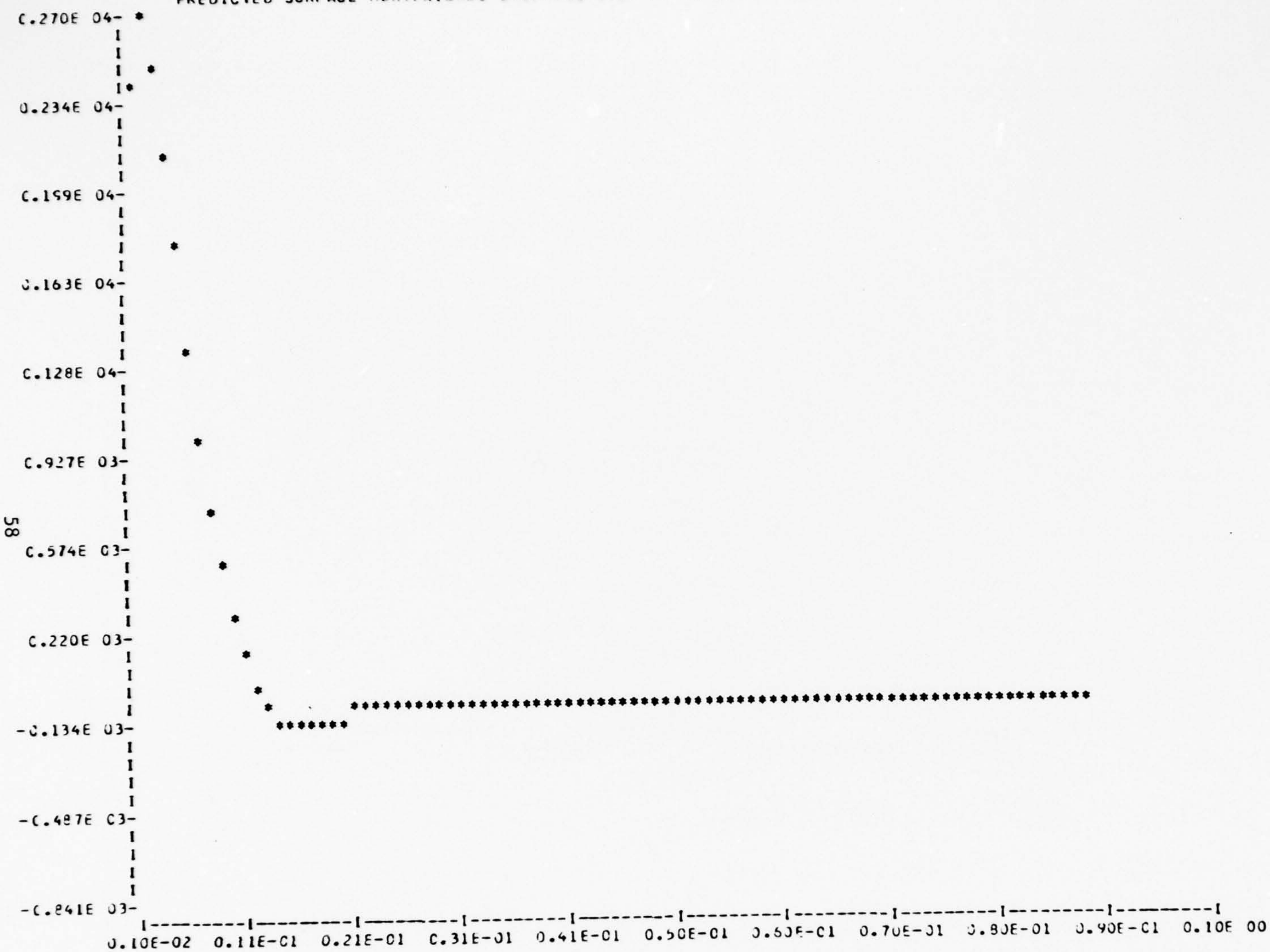
PART II Figure 5 (M60 Gun Thermocouple 10 21 Inches From Breech)



MAX. X = C.10000E 00 MAX. Y = 0.12354E 04 SCALE CN X-AXIS = 0.99)0E-02 UNITS/INCH
MIN. X = 0.10000E-02 MIN. Y = -0.31802E 04 SCALE CN Y-AXIS = 0.52 7E 03 UNITS/INCH

PART II Figure 6 (M60 Gun Thermocouple 7 15.0 Inches From Breech)

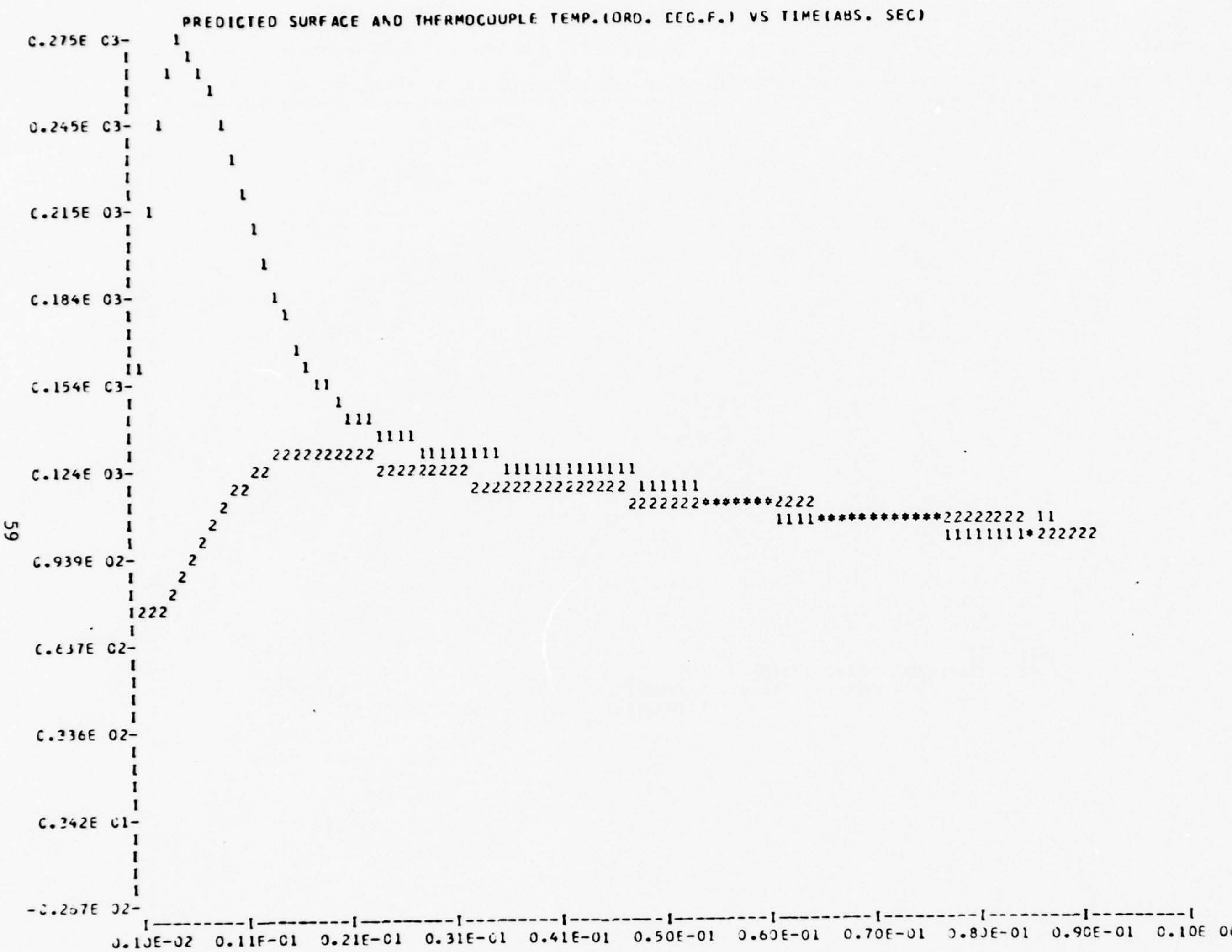
PREDICTED SURFACE HEATFX(ORD. BTU/FTSQ SEC) VS TIME(ABS. SEC)



MAX. X = 0.10000E 00 MAX. Y = 0.26952E C4 SCALE CN X-AXIS = 0.99000E-02 UNITS/INCH

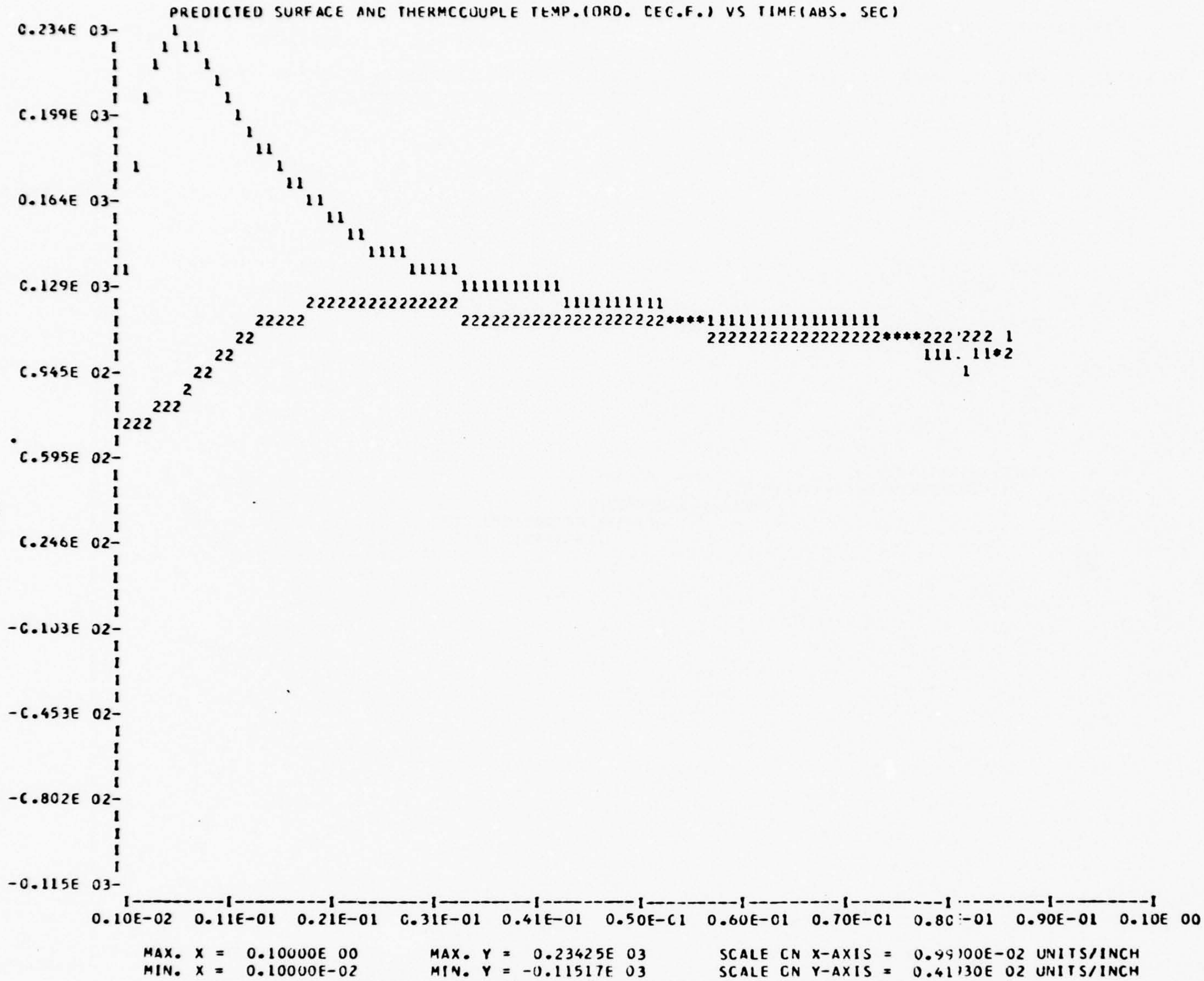
MIN. X = 0.10000E-02 MIN. Y = -0.84083E 03 SCALE CN Y-AXIS = 0.42433E 03 UNITS/INCH

PART II Figure 7 (M60 Gun Thermocouple 4 9.0 Inches From Breech)

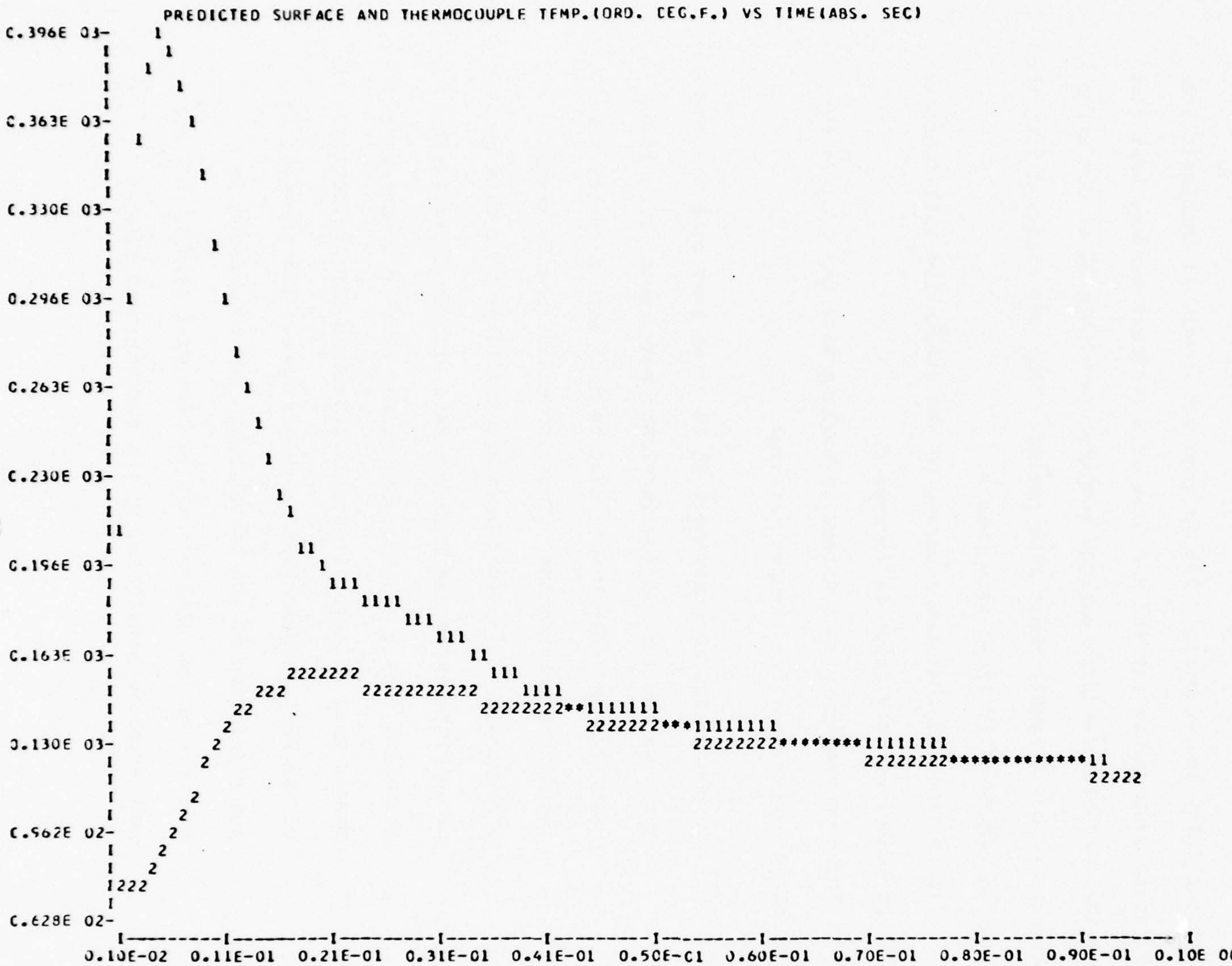


MAX. X = 0.10000E 00 MAX. Y = 0.27482E 03 SCALE ON X-AXIS = 0.99000E-02 UNITS/INCH
MIN. X = 0.10000E-02 MIN. Y = -0.26731E 02 SCALE ON Y-AXIS = 0.30136E 02 UNITS/INCH

PART II Figure 8 (M60 Gun Thermocouple 10 21 Inches From Breech)



PART II Figure 9 (M60 Gun Thermocouple 7 15.0 Inches From Breech)



MAX. X = 0.10000E 00   MAX. Y = 0.39641E 03   SCALE ON X-AXIS = 0.99000E-02 UNITS/INCH
MIN. X = 0.10000E-02   MIN. Y = 0.62848E 02   SCALE ON Y-AXIS = 0.40028E 02 UNITS/INCH

PART II Figure 10 (M60 Gun Thermocouple 4 9.0 Inches From Breech)

CONCLUSIONS AND SUGGESTIONS

The new inversion computer program was thoroughly tested for its applicability and accuracy. The program can invert an intrinsic temperature response to predict the case of a constant surface heat flux or the case of a periodic surface temperature within 2% of deviation except at the extremely short time period. This was achieved with the maximum number of 20 input data points.

It is expected that the accuracy of the prediction will increase if the number of data input is increased.

Based on the experience gained in working with the program the following suggestions are thought relevant.

(1) In conducting an experiment it is vital that both the temperature and the time in the intrinsic measurement of surface heat flux and temperature must be much more accurate than the direct measurement. This is because the inversion problem always involves predicting a large heat flux or temperature variation from the data with small variation. Therefore, for a slight error in the time or temperature measurement a large error will result in the prediction of the surface heat flux or temperature. For example, in the prediction of gun barrel heat flux considered in an error of two milliseconds in time will lead to 100 percent error in prediction of the surface heat flux.

(2) In selecting the data points for input to the computer program care must be exercised not to create a locally abrupt jump in the data. An abrupt change in data points will often introduce an abnormal fitting of a curve in its neighborhood and hence resulting in an incorrect prediction of the surface heat flux and temperature. If indeed the abrupt jump of the data must be used, then more data points in its neighborhood must also be chosen.

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APPENDIX A ANALYSIS OF THE INVERSION PROBLEM

Consider a slab, having a sufficient wall thickness, L, such that the outer surface temperature has a negligible response when the inner surface is exposed to a transient heat flux. A probe, for example a thermocouple, is located at $X = X_1$ and it is normally desirable, as reported by Chen and Li [16], to be close to the heating surface since a better transient response and more accurate experimental measurements can be obtained to reduce error amplification in the mathematical inversion program. Under these circumstances we thus assume in the analysis $L/X_1 >> 1$. The governing equation for the transient heat conduction may be written in a dimensionless form as

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} \tag{1}$$

with the initial and boundary conditions

$$\theta(x, o) = 0 \tag{2}$$

$$\theta(\infty, t) = 0 \tag{3}$$

$$\theta(1, t) = f(t) \tag{4}$$

Where $x = X/X_1$ is a dimensionless distance from the heating surface and $t = \alpha\tau/X_1^2$ is a dimensionless time or Fourier number with α being the thermal diffusivity and τ the real time. $\theta = (T - T_o)/T_o$ is the dimensionless temperature above the initially uniform temperature T_o. $f(t) = (F(\tau) - T_o)/T_o$ is the dimensionless measured temperature response at $x = 1$ with $F(\tau)$ being the measured absolute temperature. The

inversion problem is then given the interior temperature $f(t)$ to predict the surface temperature $\theta(0,t)$ and the surface heat flux per unit area $q(0,t)$ or $\left.\frac{\partial\theta}{\partial x}\right|_o = -q(0,t)X_1/T_o\kappa$. Here κ is the thermal conductivity of the solid.

The above problem may be solved by Laplace Transformation. Let the transformation be:

$$\bar{\theta}(x,s) = \int_o^\infty \theta(x,t)e^{-ts}\,dt \tag{5}$$

where θ is continuous otherwise satisfied the Dirichlet's condition. The temperature function θ is recovered by inversion of the Laplace Transformation as:

$$\theta(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \bar{\theta}e^{st}\,ds \tag{6}$$

where c is a suitable positive value. Equation (1) and the initial condition (2) under transformation (5) become:

$$\frac{d^2\bar{\theta}}{dx^2} = s\bar{\theta} \tag{7}$$

which has a solution, with integration constants A and B,

$$\bar{\theta}(x,s) = A\,e^{\sqrt{s}x} + Be^{-\sqrt{s}x} \tag{8}$$

The transformation of boundary conditions (3) and (4) into the Laplace

plane give $\overline{\theta}(\infty,s) = 0$, $\overline{\theta}(1,s) = \overline{f}(s)$. Substitution of the boundary conditions into equation (8), we get $\overline{\theta}$ and its derivative as:

$$\overline{\theta}(x,s) = \overline{f}(s)\ e^{\sqrt{s}(1 - x)} \tag{9}$$

$$-\frac{\partial\overline{\theta}}{\partial x} = \sqrt{s}\ \overline{f}(s)\ e^{\sqrt{s}(1 - x)} \tag{10}$$

According to Chen and Thomsen [14], we choose the temperature response as measured by a probe to be represented by a polynomial

$$f(t) = \sum_{n=1}^{N} b_n\ (4t)^n\ \Gamma(n+1)\ i^{2n}\ \text{erfc}\ \left(\frac{1}{2\sqrt{t}}\right) \tag{11}$$

or in Laplace plane

$$\overline{f}(s) = e^{-\sqrt{s}} \sum_{n=1}^{N} \Gamma(n + 1)\ \frac{b_n}{s^{1+n}} \tag{12}$$

The b_n's are coefficients of the expansion to be determined such that the N term polynomial describes the temperature response $f(t)$ measured at $x = 1$. With equation (12), equations (9) and (10) can be simplified to

$$\overline{\theta}(x,s) = e^{-x\sqrt{s}} \sum_{n=1}^{N} \Gamma(n + 1)\ \frac{b_n}{s^{1+n}} \tag{13}$$

$$-\frac{\partial\overline{\theta}}{\partial x} = e^{-x\sqrt{s}} \sum_{n=1}^{N} \Gamma(n + 1)\ \frac{b_n}{s^{1/2+n}} \tag{14}$$

The inversion of equations (13) and (14) at $x = 0$ give:

$$\theta(0,t) = \sum_{n=1}^{N} b_n \, t^n \tag{15}$$

$$- \left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=0} = \sum_{n=1}^{N} b_n \, t^{n-1/2} \, \frac{\Gamma(n+1)}{\Gamma(n+1/2)} \tag{16}$$

where $\theta(0,t)$ gives the surface temperature and $-\frac{\partial\theta(0,t)}{\partial x}$ gives the surface heat flux $\frac{qX_1}{\kappa T_o}$ as a function of time.

The integral of the error function in (11) is defined as:

$$i^{2n} \operatorname{erfc}\left(\frac{1}{2\sqrt{t}}\right) = \int_{\left(\frac{1}{2\sqrt{t}}\right)}^{\infty} i^{2n-1} \operatorname{erfc}(y) \, dy \tag{17}$$

with $n = 0$, $\operatorname{erfc}(y) = \frac{2}{\sqrt{\pi}} \int_{y}^{\infty} e^{-x^2} dx$. $\Gamma(n)$ in equations (11) and (16) is the gamma function or Euler's integral function of the second kind.

$$\Gamma(n) = \int_0^{\infty} e^{-\omega} \omega^{n-1} \, d\omega \tag{18}$$

It should be remarked that the choice of the particular form (11) is to ensure the convergence of the solution on the Laplace plane and an analytic inversion back to the physical plane. With b_n coefficients determined from equation (11) and the experimental measurement of the temperature response $f(t)$ at $x = 1$, the surface temperature, $T_w(t)$, is obtained from equation (15) as

$$T_w(t) = T_o \left(1 + \sum_{r=1}^{N} b_n \, t^n\right) \tag{19}$$

and the heat flux, $q(t) = \frac{-T_o \kappa}{X_1} \left. \frac{\partial \theta}{\partial x} \right|_{x=0}$, from equation (16) as:

$$q(t) = \frac{-T_o \kappa}{X_1} \sum_{n=1}^{N} b_n \, t^{n-1/2} \frac{\Gamma(n+1)}{\Gamma(n+1/2)} \tag{20}$$

The above solution is the exact solution for predicting the transient surface heat flux and temperature valid for both short and long time durations as long as the slab is thick enough such that the outer surface maintains its initial temperature. The feature of the present solution is the polynomial (11) which on the Laplace plane gives a term in equation (12) $\exp(-\sqrt{s})$ to cancel the term $\exp(\sqrt{s})$ in equations (9) and (10). This polynomial (11) as suggested by Chen and Thomsen [15] makes the present solution simpler than many inversion solutions derived in the past and valid in any time.

If the fluid temperature, $T_g(t)$, away from the surface of the slab is known, then the instantaneous heat transfer coefficients, $h(t)$, can be determined from Newton's cooling law as:

$$h(t) = \frac{q(t)}{T_g(t) - T_w(t)} \tag{21}$$

where heat flux, $q(t)$, and wall temperature $T_w(t)$ are given by equations (19) and (20), and the average heat transfer coefficient up to time t, $\bar{h}(t)$, can be defined as:

$$\bar{h}(t) = \frac{\int_0^t q(t')\, dt'}{\int_0^t [T_g(t') - T_w(t')]\, dt'} \tag{22}$$

APPENDIX B

IMPROVED COMPUTER PROGRAM

Cartesian Inversion Problem

15

```
            C I      ** DATA INPUT **                                             I
            C I      ================                                             I
            C I                                                                   I
            C I           TITLE OF DATA(BLANCK FIRST 20 SPACE)        FIRST CARD  I
            C I           BORRAD=BORE RADIUS OF CYLINDER (FT.)        SECOND CARD I
            C I           OUTR= OUTER RADIUS (FT.)                    THIRD CARD  I
            C I           THICK=OUTER RADIUS-BORRED (FT.)                         I
            C I           DIS=BORE TO THERMOCOUPLE DISTANCE (FT.)     FOURTH CARD I
            C I           TEMPO=INITIAL THERMOCOUPLE TEMPERATURE(F.)  FIFTH  CARD I
            C I           TG(1)= INITAL GAS CORE TEMPERATURE AT NORM TIME 0.00001 I
            C I                                                       SIXTH CARD  I
            C I           ALP=THERMAL DIFFUSIVITY OF MATERIAL(FTSC/SEC)SEVENTH CARDI
            C I           THCON=THERMAL CONDUCTIVITY OF MATERIAL(BTU/FT.SEC.DEG.F) I
            C I                                                       EIGHTH CARD I
            C I           NP=NUMBER OF TIME-TEMPERATURE DATA  PAIRS UP TO 25 PAIRS I
            C I                                                       NINTH CARD  I
            C I           NB= NUMBER OF B COEFF. TO BE FITTED         TENTH CARD  I
            C I           FROM 11TH  ON AR PAIRS OF TIME(I) AND TEMP(I)           I
            C I           TIME(I)=TIME FROM  THERMOCOUPLE RESPONSE CURVE(SEC.)    I
            C I           TEMP(I)= TEMPERATURE FROM THERMOCOUPLE RESPONSE (DEG. F) I
            C I           TSHFT = TIME SHIFTED FROM DATA READ IN(SEC)  LAST CARD  I
            C I___________________________________________________________________I
            C
            C
            C ____________________________________________________________________
            C I           DECLARATION STATEMENTS   START COMPUTER PROGRAM         I
            C I___________________________________________________________________I
            C
0001              IMPLICIT REAL*8(A-H,O-Z)
0002              DIMENSION        TG(210),QM(210),DTWM(210),HM(210),TGM(210),
                 1TWM(210),TOT(210),TEMO(210),HT(210),EQUR(210),DTEMR(210)
0003              DIMENSION B(26),BERFC(52),TEMP(26),TIME(26),T(26),X(26),GAM(28),
                 1C(26,26),FN(26),        G(26,210),H(26,210),ECUE(210),DTEMP(210)
                 2,TCT(26),TEMC(210),TRAN(26,26),QTQ(26,26)
0004              DATA EPS/.1D-75/
            C
            C ____________________________________________________________________
            C I           READ DATA CARDS INTO PROGRAM                            I
            C I___________________________________________________________________I
            C
0005              CALL ERRSET(208,0,-1,1,1)
0006        199   FORMAT(    20X,50H
                 1                                                   )
0007        200   FORMAT(F20.10)
0008        201   FORMAT(I2)
0009        1     READ (5,199,END=999)                                     FIRST
0010              READ (5,200) BORRAD                                      SECOND
0011              READ (5,200) OUTR                                        THIRD
0012              READ(5,200) DIS                                          FOURTH
0013              READ(5,200) TEMPO                                        FIFTH
0014              READ (5,200) TG(1)                                       SISTH
0015              READ(5,200) ALP                                          SEVENTH
0016              READ(5,20C) THCON                                        EIGHTH
0017        2     READ(5,201, END=999) NP                                  NINTH
0018        3     READ (5,201) NB                                          TENTH
0019              WRITE (6,198)
0020        198   FORMAT (1H1)
0021              WRITE (6,199)
```

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```
FORTRAN IV G LEVEL  21                MAIN              DATE = 76157          15/33/00             PAGE 0002

0022               WRITE(6,100)
0023         100   FORMAT ( 20X,' BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM')
0024               WRITE (6,109) NB
0025                  DC 10 I=1,NP
0026                  READ(5,202) TIME(I),TEMP(I)
0027          10      CONTINUE
0028               WRITE(6,107)
0029               DO 16 I=1,NP
0030          16      WRITE(6,108) TIME(I),TEMP(I)
0031         202      FORMAT(2F20.10)
0032               READ (5,200)  TSHFT
0033               WRITE (6,130) TSHFT
             C
             C ----------------------------------------------------------------
             C I            NOW THAT THE DATA HAS BEEN INPUT START CALCULATIONS      I
             C I      FIRST SETUP DATA NEEDED FOR THE CALL TC PERFC                  I
             C I      WHERE PERFC IS INTEGRATED ERROR FUNCTION                       I
             C I______________________________________________________________________I
             C
0034                             DO 5 I=1,NP
0035           5                 TIME(I) = TIME(I) - TSHFT
0036               THICK=CUTR-BORRAD
0037               RO=0.5D0
0038               GI=1.0D0
0039               TR1=ALP/(DIS**2)
             C
             C I ---------------------------------------------------------------I
             C I            CACULATE DIMENSIONLESS TIME INCREMENT  'TEMIN'          I
             C I      THE CONSTANT CRLT VARIES WITH WHAT REAL TIME RANGE WE WANT    I
             C I      BUT NOT DEPENDS ON DIFFERENT MATERIAL OR THERMOCOUPLE         I
             C I      LOCATION                                                       I
             C I ---------------------------------------------------------------I
             C
0040               CRLT=0.1D-2
0041               TEMIN=TR1*CRLT
0042               XXI=0.5D0*GI
0043               WRITE (6,203) GI,TR1
0044         203   FORMAT ('-FOR CHECKING USE  GI=',1F7.4 ,'  TR1=',1F7.4 )
0045               DO 20 I=1,NP
0046                  T(I)=TIME(I)*TR1
0047          20   X(I)=0.5D0/DSQRT(T(I))
0048          21   CONTINUE
             C
             C
             C ----------------------------------------------------------------
             C I      USE THE SUBROUTINE PERFC TO CALCULATE INTEGRATED ERROR        I
             C I      FUNCTION AND SET UP  C(I,J) MATRIX FOR THE SOLUTION OF B      I
             C I______________________________________________________________________I
             C
0049               NN1 =NB+1
0050         204   FORMAT ('-NN1 = ', I2)
0051               WRITE (6,204)  NN1
0052               DC 30 I=1,NP
0053          30      C(I,NN1)=(TEMP(I)-TEMPO )/(460.0D0+TEMPO)
0054               DO 40 I=1,NP
0055               NB2=NB*2.0D0
0056               XI = X(I)
0057               CALL PERFC (NB2,BERFC,XI)
```

/7

```
            DO 40 J=1,NB
         J2=2*J
         C(I,J)=(4.0D0*T(I))**J*PERFC(J2)*DGAMMA(J+1.0D0)
   40    CONTINUE
C
C ________________________________________________________________________
C I         NOW B COEFFICIENT FOR THE THE TEMPERATURE RESPONSE AT           I
C I         THE THERMOCOUPLE ARE SOLVED EITHER BY EXACT CR LEAST SCAURE     I
C I_______________________________________________________________________I
C
         NT=NP+1
         IF (NP - NB) 42, 43, 42
   42    CALL LTSQER (NP,NB,C,B,EPS,TIME,NT,TRAN,QTQ,TEMPO)
         GO TO 44
   43    DET = SIMUL (NP, C, B, EPS, 1,11)
   44    CONTINUE
C
C ________________________________________________________________________
C I              NOW CALCULATE HEAT FLUX FUNCTION  AND                      I
C I              CALCULATE TEMPERATURE FUNCTION                             I
C I_______________________________________________________________________I
C
         TEM = TEMIN
         DO 55 I=1,200
            DO 50 J=1,NB
         G(J,I)= TEM**J
         H(J,I)=-TEM**(J-0.5D0)*DGAMMA(J+1.0D0)/DGAMMA(J+0.5D0)
   50          CONTINUE
         TEM = TEM + TEMIN
   55       CONTINUE
C
C
C ________________________________________________________________________
C I              CALCULATE TEMPERATURE AND HEAT FLUX                        I
C I_______________________________________________________________________I
C
         DO 75 I=1,200
            DELT=0.0D0
            QUE=0.0D0
            DO 70 J=1,NB
               DELT=DELT+B(J)*G(J,I)
   70          QUE=QUE+B(J)*H(J,I)
            DTEMP(I)=DELT
         DTEMR(I)=DTEMP(I)*(460.0D0+TEMPO)+TEMPO
            EQUE(I)=-QUE
   75    EQUR(I)= EQUE(I)*(460.0D0+TEMPO)*THCON/DIS
C
C ________________________________________________________________________
C I         CALCULATE INST. AND MEAN QUANTITIES FOR TEMP. AND HEAT T.COFF.I
C I_______________________________________________________________________I
C
            HT(1)=0.0D0
            TGM(1)=0.0D0
            QM(1)=0.0D0
            DTWM(1)=0.0D0
            TWM(1)=0.0D0
            HM(1)=0.0D0
            SUMTW=0.0D0
```

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```
            SUMTG=0.0D0
            SUMQ    =0.0D0
            SUMDT=0.0D0
         TG(1)=(TG(1)-TEMPO)/(460.0D0+TEMPO)
         DO 80 I=1,200
            F=I
            SUMQ     = SUMQ     +EQUE(I)
            CM(I) = SUMQ/F
            SUMTG= SUMTG + TG(I)
            TGM(I)= SUMTG/F
            SUMDT = SUMDT + TG(I)- DTEMP(I)
            SUMTW = SUMTW + DTEMP(I)
            TWM(I)=SUMTW/F
            HM(I)= SUMQ    /SUMDT
            HT(I)=EQUE(I)/( TG(I) - DTEMP(I))
         TG(I+1)=(DTEMP(I)+DTEMP(I+1))/2.0D0+(EQUE(I)+EQUE(I+1))/(2.0D0*
     1         HT(I))
   80       CONTINUE
C
C ------------------------------------------------------------------
C I           CALCULATE THERMOCOUPLE TEMPERATURE FROM FITTED EQ.      I
C I_________________________________________________________________I
C
         TEM = TEMIN
         DO 78 I=1,200
         TEMC(I) = 0.0D0
         XX=0.5D0/DSQRT(TEM)
         CALL PERFC (NB2,BERFC,XX)
            DO 76 J=1,NB
            J2=J*2.0D0
         TCT(J)=(TEM*4.0D0)**J*BERFC(J2)*DGAMMA(J+1.0D0)
         TEMC(I) = TEMC(I) + B(J)*TCT(J)
   76       CONTINUE
         TEM = TEM + TEMIN
         TEMC(I)=TEMC(I)*(TEMPO+460.0D0)+TEMPO
   78    CONTINUE
C
C ------------------------------------------------------------------
C I           NOW OUTPUT THE DATA                                     I
C I_________________________________________________________________I
C
         WRITE (6,198)
         WRITE (6,199)
         WRITE(6,101) BORRAD
  101    FORMAT('-BORE RADIUS  (FT.) =',F10.5)
         WRITE (6,119) OUTR
  119    FORMAT  (' OUTER RADIUS (FT.)=',F10.5)
         WRITE(6,102) DIS
  102    FORMAT(' BORE TO THERMOCOUPLE  DISTANCE (FT)=',F10.6)
         WRITE(6,103) TEMPO
  103    FORMAT(' INITIAL THERMOCOUPLE TEMPERATURE (F.)=',F10.4)
         WRITE (6,301) TG(1)
  301    FORMAT (' INITIAL GAS TEMPERATURE (F.)=',F10.4)
         WRITE(6,104) ALP
  104    FORMAT (' THERMAL DIFFUSIVITY  (FTSQ/SEC) =',F15.8)
         WRITE(6,105) THCON
  105    FORMAT(' THERMAL  CONDUCTIVITY(BTU/FT.SEC.F.)=',F15.8)
         WRITE(6,106) NP
```

17

```
0140        106 FORMAT(' NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=',I2)
0141            WRITE (6,109) NB
0142            WRITE(6,107)
0143            DO 15 I=1,NP
0144        15     WRITE(6,108) TIME(I),TEMP(I)
0145            WRITE (6,130) TSHFT
0146        109 FORMAT (' NUMBER OF B(I) COFF. TO BE FITTED = ',I2)
0147        130 FORMAT(' TIME OF DATA SHIFTED BY(SEC)' ,F10.6)
0148        108    FORMAT(' ',7X,F20.10,6X,F20.10)
0149        107 FORMAT('-',13X,'TIME',5X,'TEMPERATURE')
0150            DO 41 I=1,NB
0151        229 FORMAT (' COEFFICIENTS OF B(I)=', D16.8, '      I =',I2)
0152        41  WRITE (6,229) B(I),I
0153            WRITE (6,198)
0154            WRITE (6,199)
0155            WRITE(6,113)
0156        113 FORMAT (30X,'  TEMPERATURE AND HEAT FLUX '/,30X,'  PREDICTED BY',
               1 '  JENQ-SHING CHIOU ',///,2X,
               2'(NORTM) (REALTM) (NOR.FLUX) (REAL.FLUX) (NOR.T)   (REAL.SUF.T) (
               3TH.CO.T)')
0157            WRITE (6,115)
0158        115 FORMAT(' ','(DMLESS) (SEC.)      (DMLESS) (BTU/FTSQ SEC) (DMLESS)
               1(DEG.F.) (DEG.F.)',/)
0159            TEM = TEMIN
0160            TINC=TEM
0161            TRINC= TEMIN/TR1
0162            TREM=TRINC
0163            DO 90 I=1,200
0164               WRITE (6,114) TEM,TREM,EQUE(I),EQUR(I),DTEPP(I),DTEMR(I),TEMC
               1(I)
0165        114 FORMAT (F9.6,F11.7,2(F10.4),2(F12.4),F15.8)
0166               TEM=TEM+TINC
0167               TREM=TREM+TRINC
0168        90     CONTINUE
0169            WRITE(6,113)
0170            WRITE (6,115)
0171            WRITE (6,199)
0172            GO TO 1
            C
            C
            C ___________________________________________________________
            C I   THIS IS THE END OF THE MAIN PROGRAM                      I
            C I__________________________________________________________I
            C
0173        999 WRITE(6,1000)
0174       1000 FORMAT('-ALL DATA PROCESSED-EXECUTION OF EXECUTION')
0175            DEBUG SUBCHK
0176            END
```

```
FCRTRAN IV G LEVEL  21              PERFC              DATE = 76157        15/33/00            PAGE 0001

0001            SUBROUTINE PERFC(NP,BERFC,X)
          C
          C
          C ------------------------------------------------------------
          C I            THIS SUBROUTINE CALCULATES THE REPEATED INTERGRALS  I
          C I       OF THE COMPLEMENTARY ERROR FUNCTION                      I
          C I                                                                I
          C I       NP= NUMBER OF REPEATS INTEREALS TO BE CALCULATED         I
          C I       CERFC= REAL*8 ARRAY FOR ERROR F.                         I
          C I       X= THE INITIAL VALUE FOR ERROR FUNCTION                  I
          C I                                                                I
          C I________________________________________________________________I
          C
          C
0002            IMPLICIT REAL*8(A-H,O-Z)
0003            DIMENSION BERFC(52)
0004            DATA SRIPI/ 0.56418958355/
          C
          C ------------------------------------------------------------
          C I            INITIALIZE & CALCULATE I(1)ERFC &I(2)ERFC DEPENDING  I
          C I            ON NP                                                I
          C I________________________________________________________________I
          C
0005            CALL ERRSET(208,0,-1,1,1)
0006            DO 11 I =1,50
0007            BERFC(I) =0.0D0
0008         11 CONTINUE
0009            X2=X*X
0010            BERFC1=SRIPI*DEXP(-X2)-X*DERFC(X)
0011            BERFC(1)=BERFC1
0012            IF(NP.EQ.1) RETURN
0013            BERFC2=0.25D0*((1.0D0+2.0D0*X2)*DERFC(X)-2.0D0*SRIPI*X*DEXP(-X2))
0014            BERFC(2)=BERFC2
0015            IF(NP.EQ.2)  RETURN
          C
          C ------------------------------------------------------------
          C I            NOW GO INTO DO LOOP & CALCULATE I(NP)ERFC            I
          C I________________________________________________________________I
          C
0016            DO 10 I=3,NP
0017            BERFC(I)=0.5D0/I*(BERFC1-2.0D0*X*BERFC2)
0018                BERFC1=BERFC2
0019         10     BERFC2=BERFC(I)
0020            DEBUG SUBCHK
0021            RETURN
0022            END
```

20

21

```
0001         SUBROUTINE  LTSQER (M,JJ,PHI,B,EPS,TIME,NT,TRAN,QTQ,TEMPO)
           C
           C ---------------------------------------------------------------
           C I   THIS PROGRAM CALCULATES THE LEAST SQUARES COEFFICIENTS FOR    I
           C I   MODIFIED POLYNOMIAL WHICH IS POLYNOMIAL TIMES THE INTEGRATED   I
           C I   ERROR FUCTION                                                 I
           C I                                                                 I
           C I   M IS THE NUMBER OF DATA PAIRS TO BE USED                      I
           C I   JJ IS THE NUMBER OF DEGREE TO BE FITTED                       I
           C I_________________________________________________________________I
           C
           C
0002         REAL*8 PHI(26,26),FX(26),TRAN(26,26),QTQ(26,26),X(26),DET, FPX,
            1B(26),TIME(26)
0003         CALL ERRSET(208,0,-1,1,1)
0004         LDIM=50
0005         N1=JJ+1
0006         DO 1001 I=1,M
0007         X(I)=TIME(I)
0008         FX(I)=PHI(I,N1)
0009    1001 CONTINUE
0010         ERROR =0.000
           C
           C ---------------------------------------------------------------
           C I   FIND THE TRANSPOSE OF THE PHI    MATRIX FOR LSTSQ F TTING AND  I
           C I   PERFORM THE MULTIPLICATION   AND GET JJ BY JJ MATRIX           I
           C I_________________________________________________________________I
           C
0011         CALL TRANS (PHI, TRAN,M,JJ,NT)
0012         CALL MULT (PHI,TRAN,JJ,M,QTQ,NT)
0013         DO 4 J=1,JJ
0014         QTQ(J,N1)=0.000
0015         DO 4 K=1,M
0016       4 QTQ(J,N1)=QTQ(J,N1) + TRAN(J,K)*FX(K)
0017         WRITE (6,5) (I,QTQ(I,N1),I=1,JJ)
0018       5 FORMAT (' I=',I2,'   QTQ(I,N1)=',D16.7)
           C
           C ---------------------------------------------------------------
           C I   FIND THE SOLUTION TO THE NORMAL EQUATIONS                     I
           C I_________________________________________________________________I
           C
0019         DET = SIMUL (JJ,QTQ,B,EPS,1,20)
0020         WRITE (6,100)
0021     100 FORMAT ('  LEAST SQUARE COEFFICIENTS ')
0022         WRITE (6,6) (I,B(I)   ,I=1,JJ)
0023       6 FORMAT (' I= ',I2,  '  B(I)= ', D16.7)
0024         WRITE (6,9)
0025       9 FORMAT (//' LEAST SQ OUT PUT ',//' KTH TIME',T15,'TEMP. GIVEN ',
            1T30,'FITTED', T42, 'ERROR',T55,'PER CENTAGE')
0026         DO 15 K=1,M
0027         FPX=TEMPO
0028         DO 8 I=1,JJ
0029       8 FPX=FPX + QTQ(I,N1)* PHI(K,I)
0030         FX(K)= FX(K) + TEMPO
0031         ERR = PHI(K,N1) - FPX + TEMPO
0032         IF (PHI(K,N1).NE. 0.)  PER=ERR/(PHI(K,N1) + TEMPO)*100.0
0033         IF( ABS(ERR).LE.ERROR) GO TO 15
0034         ERROR = ABS(ERR)
```

```
0035        15   WRITE (6,17) K,X(K),FX(K), FPX,ERR,PER
0036        17   FORMAT (1I3,   3(F10.5,5X),F10.6,3X,F6.2)
0037             WRITE (6,110)   ERRCR
0038       110   FORMAT ('  MAXIMUN ERROR ='  ,D16.7)
0039             DEBUG SUBCHK
0040             RETURN
0041             END

0001             SUBROUTINE TRANS (PH1,TRAN,M,N,N1)
           C
           C ------------------------------------------------------------------
           C I      THIS SUBROUTINE GIVE THE TRANSPOSE OF THE MATRIX
           C I_________________________________________________________________I
0002             REAL*8 PH1(26,26),TRAN(26,26)
0003             CALL ERRSET(208,0,-1,1,1)
0004             DO 10 J=1,N
0005             DO 10 I=1,M
0006        10   TRAN(J,I)=PH1(I,J)
0007             RETURN
0008             END


0001             SUBROUTINE MULT (PH1,TRAN,N,M,QTQ,N1)
           C
           C ------------------------------------------------------------------
           C I      THIS SUBROUTINE GIVES MULTIPLICATION OF TWO MATRIES           I
           C I_________________________________________________________________I
0002             REAL*8 PH1(26,26),TRAN(26,26),QTQ(26,26)
0003             CALL ERRSET(208,0,-1,1,1)
0004             DO 1 I=1,N
0005             DO 1 J=1,N
0006             QTQ(I,J)=0.0D0
0007             DO 1 K=1,M
0008         1   QTQ(I,J)=QTQ(I,J)+TRAN(I,K)*PH1(K,J)
0009             RETURN
0010             END
```

```
FORTRAN IV G LEVEL  21                  SIMUL               DATE = 76157          15/33/00                PAGE 0001

0001                 FUNCTION SIMUL(N,A,X,EPS,INDIC,NRC)
               C
               C ______________________________________________________________________
               C I   .....GAUSS JORDAN TECHNIQUE MAXIMUM  PIVOT                         I
               C I   ....PAGE 290 'APPLIED NUMERICAL METHODS' CARNAHAN                  I
               C I______________________________________________________________________I
               C
0002                 IMPLICIT REAL*8(A-H,O-Z)
0003                 DIMENSION IROW(60),JCOL(60),JORD(60),Y(60),A(26,26  ),X(26)
0004                 CALL ERRSET(208,0,-1,1,1)
0005                 MAX=N
0006                 IF(INDIC.GE.0) MAX=N+1
               C     ....IS N LARGER THAN 50
0007                 IF(N.LE.50) GO TO 5
0008                 WRITE (6,200)
0009                 SIMUL=0.0D0
0010                 RETURN
               C     BEGIN ELIMINATION PROCEEDURE
0011           5     DETER=1.0D0
0012                 DO 18 K=1,N
0013                 KM1=K-1
               C    .....SEARCH FO R THE PIVOT ELEMENT
0014                 PIVOT=0.0D0
0015                 DO 11 I=1,N
0016                 DO 11 J=1,N
               C    ....SCAN IROW AND JCOL ARRAYS FOR INVALID PIVOT SUBSCRIPTS
0017                 IF(K.EQ.1) GO TO 9
0018                 DO 8 ISCAN=1,KM1
0019                 DO 8 JSCAN=1,KM1
0020                 IF(I.EQ.IROW(ISCAN)) GO TO 11
0021                 IF(J.EQ.JCOL(JSCAN)) GO TO 11
0022           8     CONTINUE
0023           9     IF(DABS(A(I,J)).LE.DABS(PIVOT)) GO TO 11
0024                 PIVOT=A(I,J)
0025                 IROW(K)=I
0026                 JCOL(K)=J
0027           11     CONTINUE
               C     ....INSURE THAT SELECTED PIVOT IS LARGER THAN EPS
0028                 IF(DABS(PIVOT).GT.EPS) GO TO 13
0029                 SIMUL=0.0D0
0030                 RETURN
               C     UPDATE THE DETERMINANT VALUE...
0031           13    IROWK=IROW(K)
0032                 JCOLK=JCOL(K)
0033                 DETER=DETER*PIVOT
               C     ....NORMALIZE PIVOT ROW ELEMENTS
0034                 DO 14 J=1,MAX
0035           14     A(IROWK,J)=A(IROWK,J)/PIVOT
               C    ...CARRY OUT ELIMATION AND DEVELOPE INVERSE
0036                 A(IROWK,JCOLK)=1./PIVOT
0037                 DO 18 I=1,N
0038                 AIJCK=A(I,JCOLK)
0039                 IF(I.EQ.IROWK) GO TO 18
0040                 A(I,JCOLK)=-AIJCK/PIVOT
0041                 DO 17 J=1,MAX
0042           17    IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
0043           18      CONTINUE
               C   ....ORDER SOLUTION VALUES AND CREATE JORD ARRAY
```

```
      DO 20 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
      JORD(IROWI)=JCOLI
   20 IF(INDIC.GE.0) X(JCOLI)=A(IROWI,MAX)
C     ADJUST SIGN OF DETERMINANT
      INTCH=0
      NM1=N-1
      DO 22 I=1,NM1
      IP1=I+1
      DO 22 J=IP1,N
      IF(JORD(J).GE.JORD(I))  GO TO 22
      JTEMP=JORD(J)
      JORD(J)=JORD(I)
      JORD(I)=JTEMP
      INTCH=INTCH+1
   22 CONTINUE
      IF(INTCH/2*2.NE.INTCH) DETER=-DETER
C    ....IF INDIC IS POSITIVE RETURN WITH RESULTX
      IF(INDIC.LE.0) GO TO 26
      SIMUL=DETER
      RETURN
C     .....IF INDIC IS NEGATIVE OR ZERO, UNSCRAMBLE YHE INVERSE
C     ...FIRST BY ROWS
   26 DO 29 J=1,N
      DO 27 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
   27 Y(JCOLI)=A(IROWI,J)
      DO 28  I=1,N
   28 A(I,J)=Y(I)
C     ...THEN BY COLUMNS
      DO 30 I=1,N
      DO 29 J=1,N
      IROWJ=IROW(J)
      JCOLJ=JCOL(J)
   29 Y(IROWJ)=A(I,JCOLJ)
      DO 30 J=1,N
   30 A(I,J)=Y(J)
C    .....RETURN FOR INDIC NEGATIVE OR ZERO
      SIMUL=DETER
      RETURN
C    ....FORMAT OFR OUTPUT STATEMENT
  200 FORMAT(10HON TOO BIG)
      DEBUG SUBCHK
      END
```

APPENDIX C

NUMERICAL RESULTS

Appendix C - 1		Constant Heat Flux Case
Appendix C - 2		Periodic Surface Temperature Case
Appendix C - 3		Gun Barrel (M60) Heating

APPENDIX C - 1

```
                    FLAT STEEL PLATE EXPOSED TO  CONST HEAT FLUX

BORE RADIUS  (FT.) =    0.05089
OUTER RADIUS (FT.)=   0.09494
BORE TO THERMOCOUPLE  DISTANCE (FT)=  0.004405
INITIAL THERMOCOUPLE TEMPERATURE (F.)=   67.0000
INITIAL GAS TEMPERATURE (F.)=    4.6167
THERMAL DIFFUSIVITY  (FTSQ/SEC) =      0.00010307
THERMAL  CONDUCTIVITY(BTU/FT.SEC.F.)=       0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=10
NUMBER OF B(I) COFF. TO BE FITTED = 10

        TIME       TEMPERATURE
          0.0188260648               69.1882892400
          0.0376521296               84.0565859500
          0.0564781944              106.9027670000
          0.0753042592              132.7359606000
          0.0941303241              159.4852637000
          0.1129563889              186.2713064000
          0.1317824537              212.7038204000
          0.1506085185              238.6098830000
          0.1694345833              263.9195910000
          0.1882606481              288.6139635000
TIME OF DATA SHIFTED BY(SEC)  0.0
COEFFICIENTS OF B(I)=  0.12469107D 02       I = 1
COEFFICIENTS OF B(I)= -0.23173966D 03       I = 2
COEFFICIENTS OF B(I)=  0.25749915D 04       I = 3
COEFFICIENTS OF B(I)= -0.16462694D 05       I = 4
COEFFICIENTS OF B(I)=  0.64304700D 05       I = 5
COEFFICIENTS OF B(I)= -0.15856017D 06       I = 6
COEFFICIENTS OF B(I)=  0.24757985D 06       I = 7
COEFFICIENTS OF B(I)= -0.23710654D 06       I = 8
COEFFICIENTS OF B(I)=  0.12693355D 06       I = 9
COEFFICIENTS OF B(I)= -0.29055015D 05       I =10
```

FLAT STEEL PLATE EXPOSED TO CONST HEAT FLUX
TEMPERATURE AND HEAT FLUX
PREDICTED BY JENG-SHING CHIOU

(NORTM) (DMLESS)	(REALTM) (SEC.)	(NOR.FLUX) (DMLESS)	(REAL.FLUX) (BTU/FTSQ SEC)	(NOR.) (DMLESS)	(REAL.SUF.T) (DEG.F.)	(TH.CO.T) (DEG.F.)
0.005312	0.0010000	0.8997	597.9591	0.0601	98.6557	67.00000000
0.010624	0.0020000	1.1185	743.4044	0.1092	124.5475	67.00000000
0.015935	0.0030000	1.2076	802.6189	0.1493	145.6677	67.00000012
0.021247	0.0040000	1.2336	819.5364	0.1819	162.8741	67.00001160
0.026559	0.0050000	1.2257	814.6777	0.2085	176.9046	67.00019895
0.031871	0.0060000	1.1597	797.3942	0.2303	188.3900	67.00140603
0.037182	0.0070000	1.1652	774.4222	0.2483	197.8656	67.00591487
0.042494	0.0080000	1.1280	749.7190	0.2633	205.7826	67.01785944
0.047806	0.0090000	1.0920	725.7677	0.2761	212.5174	67.04301977
0.053118	0.0100000	1.0593	704.0854	0.2873	218.3813	67.08817495
0.058430	0.0110000	1.0314	685.5339	0.2972	223.6281	67.16032801
0.063741	0.0120000	1.0088	670.5237	0.3064	228.4624	67.26603032
0.069053	0.0130000	0.9917	659.1532	0.3151	233.0454	67.41090673
0.074365	0.0140000	0.9799	651.3071	0.3235	237.5018	67.59939341
0.079677	0.0150000	0.9730	646.7278	0.3319	241.9249	67.83465420
0.084989	0.0160000	0.9705	645.0575	0.3404	246.3816	68.11862713
0.090300	0.0170000	0.9718	645.9273	0.3490	250.9167	68.45215385
0.095612	0.0180000	0.9763	648.8856	0.3578	255.5569	68.83515341
0.100924	0.0190000	0.9833	653.5196	0.3669	260.3143	69.26681168
0.106236	0.0200000	0.9921	659.4201	0.3761	265.1892	69.74576682
0.111547	0.0210000	1.0023	666.2034	0.3855	270.1730	70.27027864
0.116859	0.0220000	1.0133	673.5161	0.3952	275.2500	70.83337522
0.122171	0.0230000	1.0247	681.0502	0.4049	280.4001	71.44797410
0.127483	0.0240000	1.0359	688.5258	0.4149	285.6000	72.09597790
0.132795	0.0250000	1.0467	695.7120	0.4247	290.8247	72.78334613
0.138106	0.0260000	1.0568	702.4168	0.4346	296.0486	73.50511570
0.143418	0.0270000	1.0660	708.4977	0.4445	301.2467	74.26058305
0.148730	0.0280000	1.0740	713.8059	0.4543	306.3953	75.04802121
0.154042	0.0290000	1.0807	718.3033	0.4639	311.3723	75.86598472
0.159354	0.0300000	1.0862	721.9203	0.4734	316.4582	76.71315524
0.164665	0.0310000	1.0903	724.6418	0.4826	321.3359	77.58836033
0.169977	0.0320000	1.0930	726.4744	0.4916	326.0913	78.49055763
0.175289	0.0330000	1.0945	727.4469	0.5004	330.7131	79.41881618
0.180601	0.0340000	1.0947	727.6067	0.5089	335.1930	80.37229643
0.185912	0.0350000	1.0938	727.0167	0.5171	339.5255	81.35023020
0.191224	0.0360000	1.0919	725.7520	0.5251	343.7082	82.35190129
0.196536	0.0370000	1.0891	723.8971	0.5327	347.7409	83.37662779
0.201848	0.0380000	1.0856	721.5428	0.5401	351.6262	84.42374619
0.207160	0.0390000	1.0815	718.7638	0.5472	355.3684	85.49255793
0.212471	0.0400000	1.0768	715.7159	0.5540	358.9742	86.58251826
0.217783	0.0410000	1.0719	712.4346	0.5606	362.4517	87.69282777
0.223095	0.0420000	1.0668	709.0324	0.5670	365.8104	88.82282619
0.228407	0.0430000	1.0616	705.5976	0.5732	369.0609	89.97178363
0.233719	0.0440000	1.0565	702.2130	0.5792	372.2146	91.13896394
0.239030	0.0450000	1.0516	698.9540	0.5850	375.2833	92.32357494
0.244342	0.0460000	1.0470	695.8885	0.5907	378.2794	93.52482037
0.249654	0.0470000	1.0428	693.0758	0.5962	381.2151	94.74187828
0.254966	0.0480000	1.0390	690.5659	0.6017	384.1024	95.97391049
0.260277	0.0490000	1.0357	688.3995	0.6071	386.9529	97.22006800
0.265589	0.0500000	1.0330	686.6077	0.6125	389.7778	98.47949701
0.270901	0.0510000	1.0309	685.2120	0.6178	392.5873	99.75134538
0.276213	0.0520000	1.0295	684.2247	0.6231	395.3909	101.03476913

```
                    CONSTANT HT FLUX

BORE RADIUS  (FT.) =    0.05089
OUTER RADIUS (FT.)=   0.09494
BORE TO THERMOCOUPLE  DISTANCE (FT)=  0.004405
INITIAL THERMOCOUPLE TEMPERATURE (F.)=   67.0000
INITIAL GAS TEMPERATURE (F.)=     4.6167
THERMAL DIFFUSIVITY  (FTSQ/SEC) =       0.00010307
THERMAL  CONDUCTIVITY(BTU/FT.SEC.F.)=      0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=20
NUMBER OF B(I) COFF. TO BE FITTED = 20

               TIME      TEMPERATURE
                 0.0188260648                69.1882892400
                 0.0376521296                84.0569859500
                 0.0564781944               106.9027670000
                 0.0753042592               132.7359606000
                 0.0941303241               159.4852637000
                 0.1129563889               186.2713064000
                 0.1317824537               212.7038204000
                 0.1506085185               238.6098830000
                 0.1694345833               263.9195910000
                 0.1882606481               288.6139635000
                 0.2070867129               312.6596465000
                 0.2259127777               336.1960294000
                 0.2447388425               359.1284574000
                 0.2635649073               381.5245816000
                 0.2823909722               403.4123851000
                 0.3012170370               424.8191289000
                 0.3200431018               445.7708137000
                 0.3388691666               466.2919338000
                 0.3576952314               486.4053559000
                 0.3765212962               506.1325321000
TIME OF DATA SHIFTED BY(SEC)  0.0
COEFFICIENTS OF B(I)=  0.13691052D 02      I = 1
COEFFICIENTS OF B(I)= -0.32663510D 03      I = 2
COEFFICIENTS OF B(I)=  0.50484596D 04      I = 3
COEFFICIENTS OF B(I)= -0.48953775D 05      I = 4
COEFFICIENTS OF B(I)=  0.32065816D 06      I = 5
COEFFICIENTS OF B(I)= -0.14950461D 07      I = 6
COEFFICIENTS OF B(I)=  0.51963638D 07      I = 7
COEFFICIENTS OF B(I)= -0.13716489D 08      I = 8
COEFFICIENTS OF B(I)=  0.28081554D 08      I = 9
COEFFICIENTS OF B(I)= -0.45136047D 08      I =10
COEFFICIENTS OF B(I)=  0.57365946D 08      I =11
COEFFICIENTS OF B(I)= -0.57805208D 08      I =12
COEFFICIENTS OF B(I)=  0.46098116D 08      I =13
COEFFICIENTS OF B(I)= -0.28898796D 08      I =14
COEFFICIENTS OF B(I)=  0.14059348D 08      I =15
COEFFICIENTS OF B(I)= -0.51954589D 07      I =16
COEFFICIENTS OF B(I)=  0.14082637D 07      I =17
COEFFICIENTS OF B(I)= -0.26396052D 06      I =18
COEFFICIENTS OF B(I)=  0.30517637D 05      I =19
COEFFICIENTS OF B(I)= -0.16408634D 04      I =20
```

AD-A041 547 IOWA INST OF HYDRAULIC RESEARCH IOWA CITY F/G 20/13
HEAT TRANSFER ANALYSIS FOR UNSTEADY HIGH VELOCITY PIPE FLOW.(U)
APR 77 C J CHEN, P LI, J S CHIOU, H Y LEE DAAG29-76-G-0123

UNCLASSIFIED RIA-R-CR-77-019 NL

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CONSTANT HT FLUX
TEMPERATURE AND HEAT FLUX
PREDICTED BY JENG-SHING CHIOU

(NORTM) (DMLESS)	(REALTM) (SEC.)	(NOR.FLUX) (DMLESS)	(REAL.FLUX) (BTU/FTSQ SEC)	(NOR.) (DMLESS)	(REAL.SUF.T) (DEG.F.)	(TM.CO.T) (DEG.F.)
0.005312	0.0010000	0.9534	633.6531	0.0642	100.8476	67.00000000
0.010624	0.0020000	1.1487	763.4597	0.1141	127.1067	67.00000000
0.015935	0.0030000	1.2077	802.7108	0.1528	147.5288	67.00000013
0.021247	0.0040000	1.2081	802.9552	0.1831	163.5185	67.00001257
0.026559	0.0050000	1.1823	785.8116	0.2072	176.1928	67.00021428
0.031871	0.0060000	1.1467	762.1832	0.2266	186.4310	67.00150476
0.037182	0.0070000	1.1102	737.8673	0.2427	194.5175	67.00628848
0.042494	0.0080000	1.0771	715.8893	0.2565	202.1776	67.01886039
0.047806	0.0090000	1.0497	697.6728	0.2687	208.6083	67.04512804
0.053118	0.0100000	1.0287	683.6940	0.2799	214.5034	67.09189289
0.058430	0.0110000	1.0139	673.8739	0.2905	220.0748	67.16603819
0.063741	0.0120000	1.0048	667.8223	0.3007	225.4707	67.27386016
0.069053	0.0130000	1.0005	664.9927	0.3108	230.7894	67.42063410
0.074365	0.0140000	1.0002	664.7808	0.3209	236.0915	67.61041071
0.079677	0.0150000	1.0029	666.5868	0.3309	241.4099	67.84599297
0.084989	0.0160000	1.0078	669.8525	0.3411	246.7570	68.12903373
0.090300	0.0170000	1.0142	674.0823	0.3513	252.1311	68.46020091
0.095612	0.0180000	1.0214	678.8529	0.3615	257.5216	68.93936949
0.100924	0.0190000	1.0288	683.8158	0.3717	262.9114	69.26581288
0.106236	0.0200000	1.0362	688.6549	0.3819	268.2820	69.73837671
0.111547	0.0210000	1.0431	693.2809	0.3921	273.6134	70.25562661
0.116859	0.0220000	1.0493	697.4246	0.4021	278.8668	70.81596707
0.122171	0.0230000	1.0547	701.0285	0.4119	284.0851	71.41773212
0.127483	0.0240000	1.0593	704.0391	0.4216	289.1936	72.05925063
0.132795	0.0250000	1.0629	706.4388	0.4311	294.2008	72.73889019
0.138106	0.0260000	1.0656	708.2383	0.4404	299.0978	73.45508363
0.143418	0.0270000	1.0674	709.4702	0.4495	303.8785	74.20634221
0.148730	0.0280000	1.0685	710.1831	0.4583	308.5397	74.99125909
0.154042	0.0290000	1.0689	710.4359	0.4669	313.0805	75.80950611
0.159354	0.0300000	1.0687	710.2940	0.4753	317.5021	76.65682648
0.164665	0.0310000	1.0680	709.8251	0.4835	321.8075	77.53502519
0.169977	0.0320000	1.0669	709.0968	0.4915	326.0012	78.44155878
0.175289	0.0330000	1.0655	708.1740	0.4992	330.0887	79.37652544
0.180601	0.0340000	1.0639	707.1171	0.5068	334.0766	80.33765615
0.185912	0.0350000	1.0622	705.9809	0.5142	337.9716	81.32430729
0.191224	0.0360000	1.0604	704.8139	0.5213	341.7811	82.33535592
0.196536	0.0370000	1.0587	703.6577	0.5285	345.5123	83.37009068
0.201848	0.0380000	1.0570	702.5469	0.5354	349.1724	84.42721940
0.207160	0.0390000	1.0555	701.5091	0.5423	352.7683	85.50585803
0.212471	0.0400000	1.0540	700.5657	0.5490	356.3066	86.60593584
0.217783	0.0410000	1.0528	699.7317	0.5556	359.7932	87.72379555
0.223095	0.0420000	1.0517	699.0165	0.5621	363.2337	88.86119521
0.228407	0.0430000	1.0508	698.4247	0.5686	366.6330	90.01631060
0.233719	0.0440000	1.0501	697.9566	0.5749	369.9955	91.18923781
0.239030	0.0450000	1.0496	697.6088	0.5813	373.3248	92.37604605
0.244342	0.0460000	1.0492	697.3750	0.5875	376.6242	93.57903028
0.249654	0.0470000	1.0490	697.2465	0.5937	379.8963	94.79621372
0.254966	0.0480000	1.0490	697.2129	0.5999	383.1430	96.02685004
0.260277	0.0490000	1.0491	697.2626	0.6060	386.3661	97.27017517
0.265589	0.0500000	1.0493	697.3833	0.6121	389.5668	98.52545874
0.270901	0.0510000	1.0495	697.5625	0.6181	392.7458	99.79200505
0.276213	0.0520000	1.0499	697.7876	0.6241	395.9038	101.06915367

```
FLAT STEEL PLATE EXPOSED TO  CONST HEAT FLUX

BORE RADIUS  (FT.) =    0.05089
OUTER RADIUS (FT.)=   0.09494
BORE TC THERMOCOUPLE  DISTANCE (FT)=  0.002203
INITIAL THERMOCOUPLE TEMPERATURE (F.)=   67.0000
INITIAL GAS TEMPERATURE (F.)=    4.6167
THERMAL DIFFUSIVITY  (FTSQ/SEC) =      0.00010307
THERMAL  CONDUCTIVITY(BTU/FT.SEC.F.)=      0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=16
NUMBER OF B(I) COFF. TO BE FITTED = 16

              TIME       TEMPERATURE
                0.0138260648                 99.8679803200
                0.0376521296                152.8049415000
                0.0564781944                201.5980147000
                0.0753042592                245.9095645000
                0.0941303241                286.5662660000
                0.1129563889                324.2767099000
                0.1317824537                359.5729166000
                0.1506085185                392.8529192000
                0.1694345833                424.4199064000
                0.1882606481                454.5097479000
                0.2070867129                483.3096407000
                0.2259127777                510.9708291000
                0.2447388425                537.6174433000
                0.2635649073                563.3527744000
                0.2823909722                588.2638204000
                0.3012170370                612.4246448000
TIME OF DATA SHIFTED BY(SEC)  0.0
COEFFICIENTS OF B(I)=  0.260445820 01        I = 1
COEFFICIENTS OF B(I)= -0.995953350 01        I = 2
COEFFICIENTS OF B(I)=  0.264640390 02        I = 3
COEFFICIENTS OF B(I)= -0.456087120 02        I = 4
COEFFICIENTS OF B(I)=  0.538452170 02        I = 5
COEFFICIENTS OF B(I)= -0.454348940 02        I = 6
COEFFICIENTS OF B(I)=  0.281926690 02        I = 7
COEFFICIENTS OF B(I)= -0.130943510 02        I = 8
COEFFICIENTS OF B(I)=  0.459406290 01        I = 9
COEFFICIENTS OF B(I)= -0.121922320 01        I =10
COEFFICIENTS OF B(I)=  0.243133610 00        I =11
COEFFICIENTS OF B(I)= -0.358315670-01        I =12
COEFFICIENTS OF B(I)=  0.378303190-02        I =13
COEFFICIENTS OF B(I)= -0.270483700-03        I =14
COEFFICIENTS OF B(I)=  0.117253270-04        I =15
COEFFICIENTS OF B(I)= -0.232574530-06        I =16
```

FLAT STEEL PLATE EXPOSED TO CONST HEAT FLUX
TEMPERATURE AND HEAT FLUX
PREDICTED BY JENG-SHING CHIOU

(NORTM) (DMLESS)	(REALTM) (SEC.)	(NOR.FLUX) (DMLESS)	(REAL.FLUX) (BTU/FTSQ SEC)	(NOR.) (DMLESS)	(REAL.SUF.T) (DEG.F.)	(TH.CO.T) (DEG.F.)
0.021247	0.0010000	0.3850	511.7559	0.0511	93.9223	67.00000252
0.042494	0.0020000	0.4909	652.5904	0.0946	116.8632	67.00421528
0.063741	0.0030000	0.5442	723.3625	0.1317	136.4065	67.06951214
0.084989	0.0040000	0.5710	759.0234	0.1635	153.1643	67.32513051
0.106236	0.0050000	0.5827	774.5318	0.1909	167.5889	67.88425188
0.127483	0.0060000	0.5853	777.9966	0.2146	180.0826	68.80119885
0.148730	0.0070000	0.5825	774.3060	0.2353	190.9866	70.08033309
0.169977	0.0080000	0.5767	766.5576	0.2535	200.5690	71.69544833
0.191224	0.0090000	0.5693	756.8054	0.2697	209.1318	73.60518047
0.212471	0.0100000	0.5615	746.3435	0.2843	216.8173	75.76313149
0.233719	0.0110000	0.5537	735.0373	0.2976	223.8137	78.12364280
0.254966	0.0120000	0.5465	726.4203	0.3098	230.2596	80.64387120
0.276213	0.0130000	0.5400	717.8032	0.3212	236.2687	83.29016188
0.297460	0.0140000	0.5344	710.3417	0.3319	241.9333	86.02842004
0.318707	0.0150000	0.5297	704.0647	0.3422	247.3279	88.83394023
0.339954	0.0160000	0.5258	699.0091	0.3520	252.5117	91.68592704
0.361201	0.0170000	0.5229	695.0444	0.3615	257.5313	94.56790327
0.382448	0.0180000	0.5206	692.0911	0.3708	262.4227	97.46707977
0.403696	0.0190000	0.5191	690.0337	0.3799	267.2136	100.37374738
0.424943	0.0200000	0.5181	688.7504	0.3889	271.9242	103.28071853
0.446190	0.0210000	0.5177	688.1193	0.3977	276.5694	106.18283212
0.467437	0.0220000	0.5176	688.0237	0.4064	281.1594	109.07652357
0.488684	0.0230000	0.5178	688.3546	0.4150	285.7009	111.95946175
0.509931	0.0240000	0.5183	689.0129	0.4235	290.1977	114.83024515
0.531178	0.0250000	0.5190	689.9103	0.4320	294.6519	117.68815310
0.552426	0.0260000	0.5198	690.9699	0.4403	299.0637	120.53294501
0.573673	0.0270000	0.5207	692.1257	0.4486	303.4325	123.36770114
0.594920	0.0280000	0.5216	693.3224	0.4568	307.7570	126.18369886
0.616167	0.0290000	0.5225	694.5150	0.4650	312.0354	128.99031848
0.637414	0.0300000	0.5233	695.6675	0.4730	316.2659	131.78597381
0.658661	0.0310000	0.5242	696.7522	0.4809	320.4467	134.56806272
0.679908	0.0320000	0.5249	697.7489	0.4888	324.5761	137.33993401
0.701156	0.0330000	0.5256	698.6440	0.4965	328.6526	140.10186698
0.722403	0.0340000	0.5262	699.4250	0.5041	332.6750	142.85106107
0.743650	0.0350000	0.5267	700.1007	0.5117	336.6427	145.59063297
0.764897	0.0360000	0.5271	700.6591	0.5191	340.5550	148.31961936
0.786144	0.0370000	0.5274	701.1078	0.5264	344.4120	151.03798356
0.807391	0.0380000	0.5277	701.4527	0.5336	348.2138	153.74562473
0.828638	0.0390000	0.5279	701.7013	0.5407	351.9610	156.44238365
0.849886	0.0400000	0.5280	701.8626	0.5477	355.6545	159.12807914
0.871133	0.0410000	0.5281	701.9464	0.5546	359.2952	161.80246558
0.892380	0.0420000	0.5281	701.9630	0.5615	362.8845	164.46531393
0.913627	0.0430000	0.5280	701.9225	0.5682	366.4238	167.11635702
0.934874	0.0440000	0.5280	701.8350	0.5748	369.9148	169.75534380
0.956121	0.0450000	0.5279	701.7103	0.5813	373.3591	172.382027[illegible]3
0.977368	0.0460000	0.5278	701.5574	0.5878	376.7585	174.99617616
0.998615	0.0470000	0.5276	701.3847	0.5941	380.1149	177.59757988
1.019863	0.0480000	0.5275	701.1999	0.600[illegible]	383.[illegible]301	180.18604955
1.041110	0.0490000	0.5274	701.0095	0.6067	385.7059	182.76143046
1.062357	0.0500000	0.5272	700.8155	0.6128	389.9641	185.32359428
1.083604	0.0510000	0.5271	700.6347	0.6189	393.1465	187.87244531
1.104851	0.0520000	0.5269	700.4594	0.6249	396.3147	190.40791979

APPENDIX C-2 THE SLAB WITH PERIODIC SURFACE TEMPERATURE

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BORE RADIUS  (FT.) =    0.05089
OUTER RADIUS (FT.)=    0.10200
BORE TO THERMOCOUPLE  DISTANCE (FT)=  0.000220
INITIAL THERMOCOUPLE TEMPERATURE (F.)=    80.0000
INITIAL GAS TEMPERATURE (F.)=     4.4815
THERMAL DIFFUSIVITY  (FTSQ/SEC) =       0.00010307
THERMAL  CONDUCTIVITY(BTU/FT.SEC.F.)=       0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=15
NUMBER OF B(I) COFF. TO BE FITTED = 15

                  TIME                       TEMPERATURE

             0.0007000000               217.4304650000
             0.0009000000               276.4284470000
             0.0011000000               335.0123870000
             0.0013000000               390.6554760000
             0.0015000000               441.2844000000
             0.0017000000               485.1659150000
             0.0019000000               520.8663420000
             0.0021000000               547.2406690000
             0.0023000000               563.4333160000
             0.0025000000               568.8819130000
             0.0027000000               563.3198170000
             0.0029000000               546.7749510000
             0.0031000000               519.5636880000
             0.0033000000               482.2791520000
             0.0035000000               435.7737270000
TIME OF DATA SHIFTED BY(SEC)  0.0
COEFFICIENTS OF B(I)=  0.51584719D 00      I = 1
COEFFICIENTS OF B(I)= -0.80435924D-01      I = 2
COEFFICIENTS OF B(I)=  0.10835798D 00      I = 3
COEFFICIENTS OF B(I)= -0.11625776D 00      I = 4
COEFFICIENTS OF B(I)=  0.79240715D-01      I = 5
COEFFICIENTS OF B(I)= -0.39279289D-01      I = 6
COEFFICIENTS OF B(I)=  0.14544315D-01      I = 7
COEFFICIENTS OF B(I)= -0.40695150D-02      I = 8
COEFFICIENTS OF B(I)=  0.86302132D-03      I = 9
COEFFICIENTS OF B(I)= -0.13791658D-03      I =10
COEFFICIENTS OF B(I)=  0.16341443D-04      I =11
COEFFICIENTS OF B(I)= -0.13918819D-05      I =12
COEFFICIENTS OF B(I)=  0.80561120D-07      I =13
COEFFICIENTS OF B(I)= -0.23364692D-08      I =14
COEFFICIENTS OF B(I)=  0.45842994D-10      I =15
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THE SLAB WITH PERIODIC SURFACE TEMPERATURE
TEMPERATURE AND HEAT FLUX
PREDICTED BY HSAI-YIN LEE

(NORTM) (DMLESS)	(REALTM) (SEC.)	(NOR.FLUX) (DMLESS)	(REAL.FLUX) (BTU/FTSQ SEC)	(NOR.) (DMLESS)	(REAL.SUF.T) (DEG.F.)	(TH.CO.T) (DEG.F.)
0.212375	0.0001000	0.2596	3534.6075	0.1069	137.6490	82.44126535
0.424750	0.0002000	0.3600	4902.0712	0.2100	193.4060	94.33066884
0.637125	0.0003000	0.4343	5914.7548	0.3111	247.5886	112.92435213
0.849500	0.0004000	0.4942	6729.3413	0.4101	301.4311	135.71658133
1.061875	0.0005000	0.5433	7398.4433	0.5065	353.5044	161.29655414
1.274250	0.0006000	0.5835	7946.5473	0.5998	403.8938	188.75535437
1.486625	0.0007000	0.6159	8387.6363	0.6894	452.2995	217.43046500
1.699000	0.0008000	0.6411	8730.7338	0.7748	498.3938	246.75555393
1.911375	0.0009000	0.6596	8982.1087	0.8554	541.9112	276.42844700
2.123749	0.0010000	0.6717	9146.3874	0.9307	582.5683	305.94233372
2.336124	0.0011000	0.6776	9227.2243	1.0002	620.1201	335.01238700
2.548499	0.0012000	0.6776	9227.7377	1.0636	654.3358	363.33990137
2.760874	0.0013000	0.6720	9150.7958	1.1204	685.0052	390.65547600
2.973249	0.0014000	0.6608	8999.2010	1.1703	711.9396	416.71232113
3.185624	0.0015000	0.6444	8775.8082	1.2129	734.9734	441.28440300
3.397999	0.0016000	0.6230	8483.6003	1.2481	753.9650	464.16499851
3.610374	0.0017000	0.5967	8125.7368	1.2756	768.7977	485.16591900
3.822749	0.0018000	0.5658	7705.5645	1.2951	779.3802	504.11707491
4.035124	0.0019000	0.5307	7226.7369	1.3068	785.6476	520.86634200
4.247499	0.0020000	0.4915	6693.0170	1.3103	787.5615	535.27955780
4.459874	0.0021000	0.4486	6108.4798	1.3053	785.1103	547.24066900
4.672249	0.0022000	0.4022	5477.4032	1.2932	776.3093	556.65176569
4.884624	0.0023000	0.3528	4804.2763	1.2726	767.2005	563.43331601
5.096999	0.0024000	0.3006	4093.7832	1.2442	751.8527	567.52422565
5.309374	0.0025000	0.2461	3350.7838	1.2081	732.3605	568.88191302
5.521749	0.0026000	0.1895	2580.2907	1.1645	708.8443	567.48231449
5.734124	0.0027000	0.1313	1787.4441	1.1139	681.4492	563.31991705
5.946498	0.0028000	0.0718	977.4862	1.0562	650.3441	556.40710970
6.158873	0.0029000	0.0114	155.7348	0.9921	615.7210	546.77495113
6.371248	0.0030000	-0.0494	-672.4449	0.9218	577.7933	534.47165521
6.583623	0.0031000	-0.1103	-1501.6767	0.8459	536.7551	519.56362830
6.795998	0.0032000	-0.1709	-2326.6171	0.7648	492.9790	502.13315545
7.008373	0.0033000	-0.2307	-3141.9700	0.6789	446.6147	482.27915268
7.220748	0.0034000	-0.2895	-3942.3326	0.5889	397.9893	460.11639292
7.433123	0.0035000	-0.3467	-4721.4837	0.4952	347.4173	435.77372846
7.645498	0.0036000	-0.4017	-5470.2210	0.3987	295.2761	409.39954805
7.857873	0.0037000	-0.4532	-6170.9074	0.3002	242.1060	381.16527502
8.070248	0.0038000	-0.4983	-6785.2335	0.2016	188.8522	351.31552388
8.282623	0.0039000	-0.5308	-7228.9695	0.1063	137.3908	320.20243324
8.494998	0.0040000	-0.5378	-7323.1104	0.0215	91.5981	288.48962607

TEMPERATURE AND HEAT FLUX
PREDICTED BY HSAI-YIN LEE

(NORTM) (REALTM) (NOR.FLUX) (REAL.FLUX) (NOR.) (REAL.SUF.T) (TH.CO.T)
(DMLESS) (SEC.) (DMLESS) (BTU/FTSQ SEC) (DMLESS) (DEG.F.) (DEG.F.)

THE SLAB WITH PERIODIC SURFACE TEMPERATURE

APPENDIX C - 3

M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREECH
TEMPERATURE AND HEAT FLUX
PREDICTED BY HSAI-YIN LEE

(NORTM) (DMLESS)	(REALTM) (SEC.)	(NOR.FLUX) (DMLESS)	(REAL.FLUX) (BTU/FTSQ SEC)	(NOR.) (DMLESS)	(REAL.SUF.T) (DEG.F.)	(TH.CO.T) (DEG.F.)
0.036957	0.0010000	0.8394	1504.5883	0.1524	160.8969	78.80248541
0.073914	0.0020000	0.9208	1650.3766	0.2540	215.6479	79.10283268
0.110872	0.0030000	0.8640	1548.6441	0.3169	249.5496	80.75107503
0.147829	0.0040000	0.7536	1350.6857	0.3509	267.8668	84.24427704
0.184786	0.0050000	0.6257	1121.5054	0.3638	274.8224	89.27332529
0.221743	0.0060000	0.4987	893.8086	0.3618	273.7626	95.23588854
0.258701	0.0070000	0.3818	684.3901	0.3499	267.3031	101.53891376
0.295658	0.0080000	0.2796	501.2017	0.3316	257.4543	107.70954560
0.332615	0.0090000	0.1936	347.0147	0.3098	245.7306	113.41640288
0.369572	0.0100000	0.1236	221.5337	0.2866	233.2437	118.45552591
0.406529	0.0110000	0.0685	122.6979	0.2635	220.7827	122.72522448
0.443487	0.0120000	0.0265	47.5114	0.2414	208.8817	126.20000000
0.480444	0.0130000	-0.0041	-7.4187	0.2210	197.8764	128.90746934
0.517401	0.0140000	-0.0254	-45.5354	0.2026	187.9522	130.90947620
0.554358	0.0150000	-0.0391	-70.1131	0.1863	179.1825	132.28740140
0.591316	0.0160000	-0.0469	-84.1269	0.1722	171.5612	133.13121584
0.628273	0.0170000	-0.0503	-90.1846	0.1600	165.0278	133.53167289
0.665230	0.0180000	-0.0505	-90.4995	0.1498	159.4880	133.57503309
0.702187	0.0190000	-0.0485	-86.8935	0.1411	154.8290	133.33976926
0.739144	0.0200000	-0.0451	-80.8193	0.1339	150.9312	132.89477680
0.776102	0.0210000	-0.0409	-73.3944	0.1278	147.6772	132.29869263
0.813059	0.0220000	-0.0365	-65.4423	0.1228	144.9571	131.60000000
0.850016	0.0230000	-0.0321	-57.5369	0.1185	142.6723	130.83766159
0.886973	0.0240000	-0.0279	-50.0464	0.1150	140.7379	130.04207940
0.923931	0.0250000	-0.0241	-43.1766	0.1119	139.0826	129.23622713
0.960888	0.0260000	-0.0206	-37.0095	0.1092	137.6490	128.43684019
0.997845	0.0270000	-0.0176	-31.5388	0.1069	136.3923	127.65558013
1.034802	0.0280000	-0.0149	-26.7007	0.1048	135.2788	126.90011703
1.071759	0.0290000	-0.0125	-22.3991	0.1030	134.2837	126.17509340
1.108717	0.0300000	-0.0103	-18.5266	0.1013	133.3900	125.48295008
1.145674	0.0310000	-0.0084	-14.9803	0.0998	132.5860	124.82460683
1.182631	0.0320000	-0.0065	-11.6736	0.0985	131.8634	124.20000000
1.219588	0.0330000	-0.0048	-8.5437	0.0973	131.2163	123.60848629
1.256546	0.0340000	-0.0031	-5.5551	0.0962	130.6394	123.04912647
1.293503	0.0350000	-0.0015	-2.7011	0.0953	130.1271	122.52086579
1.330460	0.0360000	-0.0000	-0.0017	0.0944	129.6728	122.02262934
1.367417	0.0370000	0.0014	2.5000	0.0937	129.2682	121.55335075
1.404374	0.0380000	0.0026	4.7427	0.0930	128.9033	121.11195267
1.441332	0.0390000	0.0037	6.6538	0.0924	128.5662	120.69729537
1.478289	0.0400000	0.0045	8.1551	0.0918	128.2437	120.30810907
1.515246	0.0410000	0.0051	9.1704	0.0912	127.9213	119.94292268
1.552203	0.0420000	0.0054	9.6319	0.0905	127.5841	119.60000000
1.589161	0.0430000	0.0053	9.4859	0.0899	127.2172	119.27729167
1.626118	0.0440000	0.0049	8.6978	0.0891	126.8066	118.97240901
1.663075	0.0450000	0.0040	7.2564	0.0882	126.3397	118.68262348
1.700032	0.0460000	0.0029	5.1763	0.0872	125.8062	118.40489326
1.736989	0.0470000	0.0014	2.4992	0.0861	125.1984	118.13591670
1.773947	0.0480000	-0.0004	-0.7061	0.0848	124.5119	117.87221030
1.810904	0.0490000	-0.0024	-4.3452	0.0834	123.7461	117.61020774
1.847861	0.0500000	-0.0046	-8.3012	0.0819	122.9038	117.34637497
1.884818	0.0510000	-0.0069	-12.4387	0.0802	121.9919	117.07733556
1.921776	0.0520000	-0.0093	-16.6090	0.0784	121.0208	116.80000000

(NORTM) (DMLESS)	(REALTM) (SEC.)	(NOR.FLUX) (DMLESS)	(REAL.FLUX) (BTU/FTSQ SEC)	(NOR.) (DMLESS)	(REAL.SUF.T) (DEG.F.)	(TH.CO.T) (DEG.F.)
1.958733	0.0530000	-0.0115	-20.6564	0.0765	120.0045	116.51169214
1.995690	0.0540000	-0.0136	-24.4246	0.0745	118.9596	116.21026603
2.032647	0.0550000	-0.0155	-27.7644	0.0726	117.9049	115.89420669
2.069605	0.0560000	-0.0170	-30.5408	0.0706	116.8606	115.56270890
2.106562	0.0570000	-0.0182	-32.6399	0.0688	115.8475	115.21572882
2.143519	0.0580000	-0.0190	-33.9759	0.0670	114.8858	114.85400433
2.180476	0.0590000	-0.0192	-34.4966	0.0653	113.9941	114.47904109
2.217433	0.0600000	-0.0191	-34.1882	0.0638	113.1885	114.09306290
2.254391	0.0610000	-0.0185	-33.0787	0.0625	112.4817	113.69892615
2.291348	0.0620000	-0.0174	-31.2390	0.0614	111.8818	113.30000000
2.328305	0.0630000	-0.0161	-28.7829	0.0605	111.3918	112.90001541
2.365262	0.0640000	-0.0144	-25.8643	0.0598	111.0094	112.50288784
2.402220	0.0650000	-0.0126	-22.6726	0.0593	110.7258	112.11252006
2.439177	0.0660000	-0.0108	-19.4252	0.0589	110.5267	111.73259289
2.476134	0.0670000	-0.0091	-16.3580	0.0586	110.3920	111.36635320
2.513091	0.0680000	-0.0077	-13.7137	0.0585	110.2968	111.01640954
2.550048	0.0690000	-0.0065	-11.7275	0.0583	110.2122	110.68454671
2.587006	0.0700000	-0.0059	-10.6115	0.0581	110.1071	110.37157113
2.623963	0.0710000	-0.0059	-10.5372	0.0578	109.9494	110.07719914
2.660920	0.0720000	-0.0065	-11.6178	0.0574	109.7091	109.80000000
2.697877	0.0730000	-0.0077	-13.8892	0.0567	109.3599	109.53740473
2.734835	0.0740000	-0.0096	-17.2932	0.0558	108.8823	109.28579062
2.771792	0.0750000	-0.0121	-21.6612	0.0547	108.2665	109.04064932
2.808749	0.0760000	-0.0149	-26.7011	0.0533	107.5151	108.79684356
2.845706	0.0770000	-0.0178	-31.9890	0.0517	106.6461	108.54895463
2.882663	0.0780000	-0.0206	-36.9667	0.0499	105.6949	108.29171788
2.919621	0.0790000	-0.0228	-40.9470	0.0481	104.7166	108.02053902
2.956578	0.0800000	-0.0241	-43.1290	0.0464	103.7867	107.73207721
2.993535	0.0810000	-0.0238	-42.6264	0.0449	103.0010	107.42487414
3.030492	0.0820000	-0.0215	-38.5100	0.0439	102.4737	107.10000000
3.067450	0.0830000	-0.0167	-29.8692	0.0437	102.3342	106.76167727
3.104407	0.0840000	-0.0089	-15.8947	0.0444	102.7200	106.41733289
3.141364	0.0850000	0.0022	4.0146	0.0463	103.7685	106.08051697
3.178321	0.0860000	0.0168	30.1143	0.0497	105.6030	105.76611245
3.215278	0.0870000	0.0347	62.1450	0.0548	108.3163	105.49524511
3.252236	0.0880000	0.0553	99.1241	0.0615	111.9472	105.29228652
3.289193	0.0890000	0.0776	139.0949	0.0699	116.4528	105.18432393
3.326150	0.0900000	0.0998	178.8266	0.0796	121.6722	105.19945099
3.363107	0.0910000	0.1191	213.4599	0.0900	127.2828	105.36421092
3.400065	0.0920000	0.1317	236.0913	0.1001	132.7477	105.70000001
3.437022	0.0930000	0.1324	237.2896	0.1085	137.2516	106.21821278
3.473979	0.0940000	0.1141	204.5369	0.1129	139.6254	106.91388248
3.510936	0.0950000	0.0678	121.5864	0.1104	138.2573	107.75753978
3.547893	0.0960000	-0.0180	-32.2718	0.0969	130.9886	108.68498008
3.584851	0.0970000	-0.1579	-283.0449	0.0672	114.9924	109.58459482
3.621808	0.0980000	-0.3699	-662.9825	0.0145	86.6343	110.28188452
3.658765	0.0990000	-0.6760	-1211.6418	-0.0696	41.3115	110.52073128
3.695722	0.1000000	-1.1030	-1977.0840	-0.1959	-26.7312	109.94096599

TEMPERATURE AND HEAT FLUX
PREDICTED BY HSAI-YIN LEE

M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREECH

M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH
BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.0100000000	110.6000000000
0.0200000000	124.5000000000
0.0300000000	123.1000000000
0.0400000000	119.6000000000
0.0500000000	116.9000000000
0.0600000000	114.1000000000
0.0700000000	112.5000000000
0.0800000000	110.3000000000
0.0900000000	107.6000000000
0.1000000000	106.8000000000

TIME OF DATA SHIFTED BY(SEC) -0.002000
BORE RADIUS (FT.) = 0.01250
OUTER RADIUS (FT.)= 0.04417
BORE TO THERMOCOUPLE DISTANCE (FT)= 0.001830
INITIAL THERMOCOUPLE TEMPERATURE (F.)= 78.8000
INITIAL GAS TEMPERATURE (F.)= 4.4937
THERMAL DIFFUSIVITY (FTSQ/SEC) = 0.00010307
THERMAL CONDUCTIVITY(BTU/FT.SEC.F.)= 0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=10
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.0120000000	110.6000000000
0.0220000000	124.5000000000
0.0320000000	123.1000000000
0.0420000000	119.6000000000
0.0520000000	116.9000000000
0.0620000000	114.1000000000
0.0720000000	112.5000000000
0.0820000000	110.3000000000
0.0920000000	107.6000000000
0.1020000000	106.8000000000

TIME OF DATA SHIFTED BY(SEC) -0.002000

COEFFICIENTS OF B(I)=	0.42998293D 01	I = 1
COEFFICIENTS OF B(I)=	-0.23795708D 02	I = 2
COEFFICIENTS OF B(I)=	0.64874208D 02	I = 3
COEFFICIENTS OF B(I)=	-0.10652420D 03	I = 4
COEFFICIENTS OF B(I)=	0.11235543D 03	I = 5
COEFFICIENTS OF B(I)=	-0.77590114D 02	I = 6
COEFFICIENTS OF B(I)=	0.34776113D 02	I = 7
COEFFICIENTS OF B(I)=	-0.97172330D 01	I = 8
COEFFICIENTS OF B(I)=	0.15343023D 01	I = 9
COEFFICIENTS OF B(I)=	-0.10434213D 00	I =10

M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH
TEMPERATURE AND HEAT FLUX
PREDICTED BY HSAI-YIN LEE

(NORTM) (DMLESS)	(REALTM) (SEC.)	(NOR.FLUX) (DMLESS)	(REAL.FLUX) (BTU/FTSQ SEC)	(NOR.) (DMLESS)	(REAL.SUF.T) (DEG.F.)	(TH.CO.T) (DEG.F.)
0.030777	0.0010000	0.6763	1106.1522	0.1116	138.9277	78.80037178
0.061555	0.0020000	0.7553	1235.3923	0.1882	180.2072	78.89254605
0.092332	0.0030000	0.7260	1187.5234	0.2382	207.1313	79.56894741
0.123109	0.0040000	0.6538	1069.4664	0.2682	223.3065	81.26193308
0.153886	0.0050000	0.5665	926.6396	0.2836	231.5973	83.98321767
0.184664	0.0060000	0.4779	781.6665	0.2885	234.2521	87.49324244
0.215441	0.0070000	0.3950	646.0729	0.2862	233.0109	91.48181303
0.246218	0.0080000	0.3211	525.3048	0.2791	229.1991	95.66106957
0.276995	0.0090000	0.2576	421.3085	0.2691	223.8069	99.80078458
0.307773	0.0100000	0.2042	334.0054	0.2575	217.5565	103.73480391
0.338550	0.0110000	0.1603	262.1869	0.2453	210.9596	107.35437364
0.369327	0.0120000	0.1248	204.0755	0.2330	204.3639	110.60000000
0.400105	0.0130000	0.0964	157.6791	0.2212	197.9933	113.44519669
0.430882	0.0140000	0.0740	121.0127	0.2101	191.9790	115.89130675
0.461659	0.0150000	0.0564	92.2309	0.1997	186.3861	117.95651340
0.492436	0.0160000	0.0426	69.7006	0.1901	181.2338	119.66963617
0.523214	0.0170000	0.0318	52.0336	0.1813	176.5111	121.06507221
0.553991	0.0180000	0.0233	38.0914	0.1733	172.1898	122.17914841
0.584768	0.0190000	0.0165	26.9727	0.1660	168.2326	123.04762728
0.615545	0.0200000	0.0110	17.9899	0.1592	164.6001	123.70412166
0.646323	0.0210000	0.0065	10.6397	0.1530	161.2549	124.17920042
0.677100	0.0220000	0.0028	4.5708	0.1473	158.1641	124.50000000
0.707877	0.0230000	-0.0003	-0.4481	0.1420	155.3006	124.69018933
0.738654	0.0240000	-0.0028	-4.5583	0.1371	152.6436	124.77016617
0.769432	0.0250000	-0.0048	-7.8388	0.1325	150.1780	124.75738986
0.800209	0.0260000	-0.0063	-10.3301	0.1282	147.8935	124.66677881
0.830986	0.0270000	-0.0074	-12.0536	0.1243	145.7836	124.51112060
0.861764	0.0280000	-0.0080	-13.0274	0.1207	143.8440	124.30145876
0.892541	0.0290000	-0.0081	-13.2779	0.1174	142.0720	124.04743335
0.923318	0.0300000	-0.0079	-12.8478	0.1144	140.4646	123.75756275
0.954095	0.0310000	-0.0072	-11.8000	0.1118	139.0180	123.43946213
0.984873	0.0320000	-0.0062	-10.2193	0.1094	137.7265	123.10000000
1.015650	0.0330000	-0.0050	-8.2111	0.1072	136.5822	122.74539846
1.046427	0.0340000	-0.0036	-5.8979	0.1054	135.5745	122.38128553
1.077204	0.0350000	-0.0021	-3.4145	0.1037	134.6903	122.01270981
1.107982	0.0360000	-0.0006	-0.9020	0.1023	133.9141	121.64412808
1.138759	0.0370000	0.0009	1.4986	0.1010	133.2281	121.27937668
1.169536	0.0380000	0.0022	3.6537	0.0999	132.6132	120.92163655
1.200314	0.0390000	0.0033	5.4431	0.0988	132.0494	120.57340112
1.231091	0.0400000	0.0041	6.7664	0.0978	131.5166	120.23645429
1.261868	0.0410000	0.0046	7.5481	0.0969	130.9956	119.91186465
1.292645	0.0420000	0.0047	7.7416	0.0959	130.4685	119.60000000
1.323423	0.0430000	0.0045	7.3318	0.0949	129.9200	119.30056472
1.354200	0.0440000	0.0039	6.3368	0.0938	129.3377	119.01266071
1.384977	0.0450000	0.0029	4.8074	0.0926	128.7128	118.73487111
1.415754	0.0460000	0.0017	2.8256	0.0914	128.0407	118.46536456
1.446532	0.0470000	0.0003	0.5012	0.0901	127.3208	118.20201648
1.477309	0.0480000	-0.0012	-2.0324	0.0886	126.5572	117.94254276
1.508086	0.0490000	-0.0028	-4.6242	0.0872	125.7579	117.68464069
1.538864	0.0500000	-0.0043	-7.1125	0.0856	124.9352	117.42613098
1.569641	0.0510000	-0.0057	-9.3323	0.0841	124.1045	117.16509487
1.600418	0.0520000	-0.0068	-11.1238	0.0826	123.2840	116.90000000

1.631155	0.0530000	-0.0075	-12.3407	0.0811	122.4936	116.62980900
1.661973	0.0540000	-0.0079	-12.8587	0.0797	121.7543	116.35406521
1.692750	0.0550000	-0.0077	-12.5835	0.0785	121.0865	116.07295073
1.723527	0.0560000	-0.0070	-11.4581	0.0774	120.5091	115.78731264
1.754304	0.0570000	-0.0058	-9.4691	0.0765	120.0384	115.49855460
1.785082	0.0580000	-0.0041	-6.6513	0.0759	119.6867	115.20909215
1.815859	0.0590000	-0.0019	-3.0911	0.0755	119.4612	114.92127132
1.846636	0.0600000	0.0007	1.0727	0.0753	119.3632	114.63825189
1.877413	0.0610000	0.0035	5.6503	0.0753	119.3871	114.36335794
1.908191	0.0620000	0.0064	10.4044	0.0756	119.5200	114.10000000
1.938968	0.0630000	0.0092	15.0566	0.0760	119.7415	113.85147476
1.969745	0.0640000	0.0118	19.2967	0.0765	120.0235	113.62074965
2.000523	0.0650000	0.0139	22.7950	0.0771	120.3310	113.41024106
2.031300	0.0660000	0.0154	25.2162	0.0776	120.6229	113.22159622
2.062077	0.0670000	0.0160	26.2372	0.0780	120.8533	113.05548585
2.092854	0.0680000	0.0156	25.5661	0.0783	120.9735	112.91144729
2.123632	0.0690000	0.0140	22.9636	0.0782	120.9340	112.73770657
2.154409	0.0700000	0.0112	18.2652	0.0777	120.6875	112.68113169
2.185186	0.0710000	0.0070	11.4040	0.0768	120.1918	112.58718931
2.215963	0.0720000	0.0015	2.4333	0.0754	119.4133	112.50000000
2.246741	0.0730000	-0.0052	-8.4524	0.0734	118.3307	112.41247419
2.277518	0.0740000	-0.0128	-20.8992	0.0708	116.9386	112.31654053
2.308295	0.0750000	-0.0210	-34.3797	0.0677	115.2512	112.20347218
2.339073	0.0760000	-0.0295	-48.1849	0.0640	113.3062	112.06431290
2.369850	0.0770000	-0.0376	-61.4238	0.0601	111.1669	111.89040074
2.400627	0.0780000	-0.0446	-73.0337	0.0559	108.9251	111.67338156
2.431404	0.0790000	-0.0500	-81.8020	0.0518	106.7017	111.40890207
2.462182	0.0800000	-0.0528	-86.4042	0.0480	104.6464	111.09135344
2.492959	0.0810000	-0.0522	-85.4602	0.0448	102.9352	110.72065125
2.523736	0.0820000	-0.0474	-77.6121	0.0426	101.7655	110.30000000
2.554513	0.0830000	-0.0377	-61.6278	0.0418	101.3482	109.83720247
2.585291	0.0840000	-0.0223	-36.5344	0.0429	101.8957	109.34525107
2.616068	0.0850000	-0.0011	-1.7851	0.0460	103.6061	108.84272839
2.646845	0.0860000	0.0260	42.5338	0.0517	106.6418	108.35392925
2.677623	0.0870000	0.0584	95.4527	0.0600	111.1017	107.90859956
2.708400	0.0880000	0.0946	154.8176	0.0709	116.9867	107.54117322
2.739177	0.0890000	0.1326	216.9337	0.0842	124.1565	107.28935705
2.769954	0.0900000	0.1688	276.1452	0.0993	132.2773	107.19191523
2.800732	0.0910000	0.1983	324.3462	0.1150	140.7586	107.28545342
2.831509	0.0920000	0.2142	350.4135	0.1297	148.6775	107.60000000
2.862286	0.0930000	0.2076	339.5534	0.1408	154.6894	108.15314365
2.893063	0.0940000	0.1666	272.5553	0.1450	156.9235	108.94246036
2.923841	0.0950000	0.0764	124.9405	0.1375	152.8606	109.93593128
2.954618	0.0960000	-0.0819	-134.0018	0.1121	139.1916	111.06001889
2.985395	0.0970000	-0.3315	-542.3027	0.0610	111.6550	112.18503324
3.016172	0.0980000	-0.7009	-1146.5363	-0.0259	64.8502	113.10738105
3.046950	0.0990000	-1.2247	-2003.1892	-0.1611	-7.9752	113.52824958
3.077727	0.1000000	-1.9442	-3100.1900	-0.3600	-115.1663	113.02823602

TEMPERATURE AND HEAT FLUX
PREDICTED BY HSAI-YIN LEE

(NORTM)	(REALTM)	(NOR.FLUX)	(REAL.FLUX)	(NOR.)	(REAL.SUF.T)	(TH.CO.T)
(DMLESS)	(SEC.)	(DMLESS)	(BTU/FTSQ SEC)	(DMLESS)	(DEG.F.)	(DEG.F.)

M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH

M60 GUN THERMOCOUPLE 4 9.0 INCHES FROM BREECH
BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.0100000000	143.2000000000
0.0200000000	157.3000000000
0.0300000000	151.0000000000
0.0400000000	144.4000000000
0.0500000000	137.5000000000
0.0600000000	132.8000000000
0.0700000000	129.0000000000
0.0800000000	125.9000000000
0.0900000000	122.8000000000
0.1000000000	119.6000000000

TIME OF DATA SHIFTED BY(SEC) -0.002000
BORE RADIUS (FT.) = 0.01250
OUTER RADIUS (FT.)= 0.05000
BORE TO THERMOCOUPLE DISTANCE (FT)= 0.001830
INITIAL THERMOCOUPLE TEMPERATURE (F.)= 78.8000
INITIAL GAS TEMPERATURE (F.)= 4.4937
THERMAL DIFFUSIVITY (FTSQ/SEC) = 0.00010307
THERMAL CONDUCTIVITY(BTU/FT.SEC.F.)= 0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=10
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.0120000000	143.2000000000
0.0220000000	157.3000000000
0.0320000000	151.0000000000
0.0420000000	144.4000000000
0.0520000000	137.5000000000
0.0620000000	132.8000000000
0.0720000000	129.0000000000
0.0820000000	125.9000000000
0.0920000000	122.8000000000
0.1020000000	119.6000000000

TIME OF DATA SHIFTED BY(SEC) -0.002000

COEFFICIENTS OF B(I)= 0.97265186D 01	I = 1
COEFFICIENTS OF B(I)= -0.56818786D 02	I = 2
COEFFICIENTS OF B(I)= 0.15078051D 03	I = 3
COEFFICIENTS OF B(I)= -0.22868973D 03	I = 4
COEFFICIENTS OF B(I)= 0.21619564D 03	I = 5
COEFFICIENTS OF B(I)= -0.13191471D 03	I = 6
COEFFICIENTS OF B(I)= 0.52022149D 02	I = 7
COEFFICIENTS OF B(I)= -0.12814655D 02	I = 8
COEFFICIENTS OF B(I)= 0.17933680D 01	I = 9
COEFFICIENTS OF B(I)= -0.10887593D 00	I =10

M60 GUN THERMOCOUPLE 4 9.0 INCHES FROM BREECH
TEMPERATURE AND HEAT FLUX
PREDICTED BY HSAI-YIN LEE

(NORTM) (DMLESS)	(REALTM) (SEC.)	(NOR.FLUX) (DMLESS)	(REAL.FLUX) (BTU/FTSQ SEC)	(NOR.) (DMLESS)	(REAL.SUF.T) (DEG.F.)	(TH.CO.T) (DEG.F.)
0.030777	0.0010000	1.5068	2464.6174	0.2497	213.3551	78.80083965
0.061555	0.0020000	1.6478	2695.2439	0.4155	302.6683	79.00826216
0.092332	0.0030000	1.5392	2517.6782	0.5171	357.4214	80.52178452
0.123109	0.0040000	1.3339	2181.8196	0.5708	386.3264	84.27817903
0.153886	0.0050000	1.0978	1795.6945	0.5895	396.4145	90.24714983
0.184664	0.0060000	0.8645	1414.0284	0.5837	393.2918	97.83164519
0.215441	0.0070000	0.6512	1065.1228	0.5616	381.3640	106.28706625
0.246218	0.0080000	0.4661	762.4626	0.5294	364.0339	114.93619058
0.276995	0.0090000	0.3123	510.7522	0.4920	343.8737	123.24990929
0.307773	0.0100000	0.1892	309.4211	0.4528	322.7744	130.86054314
0.338550	0.0110000	0.0946	154.8016	0.4144	302.0751	137.54545604
0.369327	0.0120000	0.0254	41.5359	0.3784	282.6736	143.20000000
0.400105	0.0130000	-0.0223	-36.4968	0.3458	265.1212	147.80889574
0.430882	0.0140000	-0.0523	-85.5691	0.3172	249.7018	151.42011603
0.461659	0.0150000	-0.0683	-111.7083	0.2927	236.4994	154.12282411
0.492436	0.0160000	-0.0736	-120.4482	0.2722	225.4530	156.02968789
0.523214	0.0170000	-0.0713	-116.6880	0.2554	216.4023	157.26331105
0.553991	0.0180000	-0.0640	-104.6240	0.2419	209.1240	157.94627789
0.584768	0.0190000	-0.0536	-87.7361	0.2312	203.3609	158.19423324
0.615545	0.0200000	-0.0421	-68.8098	0.2228	198.8445	158.11142795
0.646323	0.0210000	-0.0306	-49.9833	0.2162	195.3114	157.78820931
0.677100	0.0220000	-0.0201	-32.8107	0.2111	192.5163	157.30000000
0.707877	0.0230000	-0.0112	-18.3334	0.2068	190.2392	156.70737747
0.738654	0.0240000	-0.0044	-7.1563	0.2032	188.2913	156.05693060
0.769432	0.0250000	0.0003	0.4764	0.1999	186.5168	155.38263042
0.800209	0.0260000	0.0028	4.6083	0.1967	184.7935	154.70750441
0.830986	0.0270000	0.0034	5.5020	0.1935	183.0314	154.04544980
0.861764	0.0280000	0.0022	3.5769	0.1900	181.1703	153.40306061
0.892541	0.0290000	-0.0004	-0.6465	0.1863	179.1763	152.78137602
0.923318	0.0300000	-0.0040	-6.5945	0.1823	177.0384	152.17748532
0.954095	0.0310000	-0.0084	-13.6808	0.1781	174.7637	151.58594730
0.984873	0.0320000	-0.0130	-21.3395	0.1737	172.3743	151.00000000
1.015650	0.0330000	-0.0178	-29.0511	0.1691	169.9022	150.41255172
1.046427	0.0340000	-0.0222	-36.3615	0.1644	167.3862	149.81695499
1.077204	0.0350000	-0.0262	-42.8952	0.1597	164.8685	149.20757430
1.107982	0.0360000	-0.0296	-48.3621	0.1551	162.3913	148.58016428
1.138759	0.0370000	-0.0321	-52.5605	0.1507	159.9947	147.93207954
1.169536	0.0380000	-0.0339	-55.3751	0.1465	157.7144	147.26233979
1.200314	0.0390000	-0.0347	-56.7716	0.1425	155.5802	146.57157537
1.231091	0.0400000	-0.0347	-56.7889	0.1389	153.6153	145.86187822
1.261868	0.0410000	-0.0339	-55.5290	0.1356	151.8353	145.13658279
1.292645	0.0420000	-0.0325	-53.1448	0.1326	150.2481	144.40000000
1.323423	0.0430000	-0.0305	-49.8284	0.1300	148.8545	143.65712497
1.354200	0.0440000	-0.0280	-45.7978	0.1278	147.6482	142.91333763
1.384977	0.0450000	-0.0252	-41.2845	0.1259	146.6170	142.17411192
1.415754	0.0460000	-0.0223	-36.5215	0.1242	145.7437	141.44474732
1.446532	0.0470000	-0.0194	-31.7327	0.1229	145.0071	140.73013289
1.477309	0.0480000	-0.0166	-27.1228	0.1217	144.3834	140.03455198
1.508086	0.0490000	-0.0140	-22.8698	0.1207	143.8474	139.36153249
1.538864	0.0500000	-0.0117	-19.1187	0.1198	143.3738	138.71374553
1.569641	0.0510000	-0.0098	-15.9770	0.1190	142.9382	138.09295266
1.600418	0.0520000	-0.0083	-13.5124	0.1183	142.5183	137.50000000

(NORTM) (DMLESS)	(REALTM) (SEC.)	(NOR.FLUX) (DMLESS)	(REAL.FLUX) (BTU/FTSQ SEC)	(NOR.) (DMLESS)	(REAL.SUF.T) (DEG.F.)	(TH.CO.T) (DEG.F.)
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1.785082	0.0580000	-0.0073	-11.9555	0.1127	139.5111	134.47568749
1.815859	0.0590000	-0.0080	-13.1020	0.1115	138.8794	134.04054543
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1.938968	0.0630000	-0.0104	-17.0909	0.1065	136.1964	132.40068430
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3.077727	0.1000000	-0.5140	-840.8277	-0.0296	62.8478	121.55763221

TEMPERATURE AND HEAT FLUX
PREDICTED BY HSAI-YIN LEE

M60 GUN THERMOCOUPLE 4 9.0 INCHES FROM BREECH

M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREECH
BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.0100000000	126.2000000000
0.0200000000	131.6000000000
0.0300000000	124.2000000000
0.0400000000	119.6000000000
0.0500000000	116.8000000000
0.0600000000	113.3000000000
0.0700000000	109.8000000000
0.0800000000	107.1000000000
0.0900000000	105.7000000000
0.1000000000	104.2000000000

TIME OF DATA SHIFTED BY(SEC) -0.002000
BORE RADIUS (FT.) = 0.01250
OUTER RADIUS (FT.)= 0.03562
BORE TO THERMOCOUPLE DISTANCE (FT)= 0.001670
INITIAL THERMOCOUPLE TEMPERATURE (F.)= 78.8000
INITIAL GAS TEMPERATURE (F.)= 4.4937
THERMAL DIFFUSIVITY (FTSQ/SEC) = 0.00010307
THERMAL CONDUCTIVITY(BTU/FT.SEC.F.)= 0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=10
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.0120000000	126.2000000000
0.0220000000	131.6000000000
0.0320000000	124.2000000000
0.0420000000	119.6000000000
0.0520000000	116.8000000000
0.0620000000	113.3000000000
0.0720000000	109.8000000000
0.0820000000	107.1000000000
0.0920000000	105.7000000000
0.1020000000	104.2000000000

TIME OF DATA SHIFTED BY(SEC) -0.002000
COEFFICIENTS OF B(I)= 0.49358586D 01 I = 1
COEFFICIENTS OF B(I)= -0.23881322ID 02 I = 2
COEFFICIENTS OF B(I)= 0.53463543D 02 I = 3
COEFFICIENTS OF B(I)= -0.69864894D 02 I = 4
COEFFICIENTS OF B(I)= 0.57893886D 02 I = 5
COEFFICIENTS OF B(I)= -0.31318033D 02 I = 6
COEFFICIENTS OF B(I)= 0.11016609D 02 I = 7
COEFFICIENTS OF B(I)= -0.24257631D 01 I = 8
COEFFICIENTS OF B(I)= 0.30329015D 00 I = 9
COEFFICIENTS OF B(I)= -0.16413678D-01 I =10

APPENDIX D THE CASE OF OSCILATORY SURFACE TEMPERATURE

Consider a slab with a sufficient thickness, ℓ, such that when a surface is subjected to a periodic surface temperature variation with a frequency w the other surface is held at the initial temperature T_o. If properties are assumed constant the governing equation for the problem can be written as

$$\frac{\partial \nu}{\partial t} = \alpha \frac{\partial^2 \nu}{\partial x^2} \tag{1}$$

with initial and boundary conditions as

$$\nu(x, o) = 0 \tag{2}$$

$$\nu(\ell, t) = \sin wt \tag{3}$$

$$\nu(o, t) = 0 \tag{4}$$

where $\nu = (T - T_o)/(T_{max} - T_o)$ and α is the thermal diffusivity.

The solution of the problem according to Carslaw and Jaeger [4] can be written as

$$\nu = \frac{2\alpha\pi}{\ell^2} \sum_{n=1}^{\infty} (-1)(-1)^n n e^{-\alpha n^2 \pi^2 \tau/\ell^2} \sin\left(\frac{n\pi x}{\ell}\right) \int_o^t e^{\alpha n^2 \pi^2 \lambda/\ell^2} \sin w\lambda d\lambda \tag{5}$$

with

$$\int e^{ax} \sin bx\, dx = e^{ax}(a \sin bx - b \cos bx)/(a^2 + b^2) \tag{6}$$

we have

$$\nu = 2\alpha\pi \sum_{n=1}^{\infty} (-1)(-1)^n n \left[(\alpha n^2\pi^2 \sin wt - w\ell^2 \cos wt) + w\ell^2 e^{-\alpha n^2\pi^2 t/\ell^2}\right]$$

$$\cdot \sin\left(\frac{n\pi x}{\ell}\right) / [\alpha^2 n^4 \pi^4 + w^2 \ell^2] \qquad (7)$$

PART III

PREDICTION OF TRANSIENT SURFACE HEAT FLUX AND TEMPERATURE ON A HOLLOW CYLINDER

INTRODUCTION

In the study of transient heat transfer many efforts have been made on the so-called "inverse problem" [1,2] where a surface heat flux and temperature is to be predicted by the measured data at some location interior to a body.

In the previous works [1-6] the solution is represented in either an integral form after some manipulation of the contour integral from the inverse transform, or in a series form after the expansion of the solution for small and large times. Using Laplace transformation Chen and Thomsen [6] introduced a polynomial in terms of the error function to represent the response of thermocouple measurement and the inversion is accomplished for any transient surface heat flux at the inner surface of a cylindrical tube. However, their inversion solution is valid only for a short duration due to the asymptotic expansion of the modified Bessel function in the inverse Laplace transform. In this study an exact solution obtained from the inverse Laplace transform by the convolution method is given for the case of hollow cylinder. The solution is valid for both constant and variable heat flux and for both short and long time duration.

ANALYSIS

Consider a long hollow cylinder with sufficient wall thickness such that the outer surface temperature has a negligible response when the inner surface is exposed to a thermal pulse of a transient process. This condition considerably simplifies the theoretical analysis as the outer boundary may be assumed to be infinite, and only one interior probe of the cylinder

is required in the experimental measurement. The material of the cylinder is considered to be homogeneous and isotropic with constant thermal diffusivity, α. Let R_i and R_o be, respectively, the inner and outer surface radii. R_1 the radius of the probe location and t the dimensionless time. If the temperature of the cylinder is initially uniform at T_o, the mathematical problem governing the temperature T, may be written as

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} \qquad 1 < r < r_o = \infty \tag{1}$$

$$\theta\ (r,\ o) = 0 \tag{2}$$

$$\theta\ (\infty,\ t) = 0 \tag{3}$$

$$\theta\ (r_1,\ t) = f(t) \qquad 1 < r_1 < \infty \tag{4}$$

where $\theta = T - T_o$, $r = R/R_i$, $t = \alpha\tau/R_i^2$, and f(t) is the interior temperature response of the thermocouple measured at $r = r_1$ at the dimensionless time t. The problem is to predict the surface temperature θ (1, t) and heat flux per unit area

$$q = -\ (K/R_1)(\partial\theta/\partial r)\Big|_{r\,=\,1} \tag{5}$$

where K is the thermal conductivity.

The problem can be solved by Laplace transformation. Let the transformation be

$$\bar{\theta}(r,\ s) = \int_o^\infty \theta e^{-ts} dt \tag{6}$$

When θ satisfies the Dirichlet's condition the temperature function θ is recovered by inversion of the Laplace transformation as

$$\theta(r, t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \bar{\theta} e^{st} ds \tag{7}$$

where c is a suitable positive value. Equation (1) and (2) under transformation (6) becomes

$$\frac{d^2\bar{\theta}}{dr^2} + \frac{1}{r}\frac{d\bar{\theta}}{dr} = s\bar{\theta}$$

which has a solution of the form

$$\bar{\theta} = AI_o(pr) + BK_o(pr) \tag{8}$$

where I_o and K_o are modified Bessel functions of the first and second kind with $p = (s)^{1/2}$. With the boundary conditions (3) and (4), (8) becomes

$$\bar{\theta} = \bar{f}(s)\ [K_o(pr)/K_o(pr_1)] \tag{9}$$

where $\bar{f}(s)$ is the Laplace transform of the boundary condition (4).

The temperature response measured at $r = r_1$ can be expressed by a polynomial or numerous other suitable functions. In the present analysis, for reasons to be explained later, f(t) will be represented as

$$f(t) = \sum_{n=1}^{N} b_n \int_0^t F_1(\tau)\ Fn(t - \tau)d\tau \tag{10}$$

If we choose $F_1(t) = \frac{1}{2\tau} e^{-\frac{r_1^2}{4\tau}}$ and $Fn(t - \tau)$ being any arbitrary function

depending on n, for example $(t - \tau)^n$ etc. then the Laplace transform of Eq. (10) gives

$$\bar{f}(s) = \sum_{n=1}^{N} b_n K_o(pr_1)\bar{F}n(s) \tag{11}$$

where $\bar{F}n(s)$ is the Laplace transform of Fn(t). Substituting Eq. (11) into Eq. (19) we have

$$\bar{\theta}(s, r) = \sum_{n=1}^{N} b_n K_o(pr)\bar{F}n(s) \tag{12}$$

It is noted that the $K_o(pr_1)$ in Eq. (9) has been cancelled by this substitution which explains the choice of $F\ (t) = \frac{1}{4K} e^{-\frac{r_1^2}{4K}}$ in Eq. (10). The inversion of Eq. (12) gives

$$\theta(t, r) = \sum_{n=1}^{N} b_n \int_o^t \frac{1}{2\tau} e^{-\frac{r^2}{4\tau}} Fn(t - \tau)d\tau \tag{13}$$

At surface r = 1

$$\theta(t, 1) = \sum_{n=1}^{N} b_n \int_o^t \frac{1}{2\tau} e^{-\frac{1}{4\tau}} Fn(t - \tau)d\tau \tag{14}$$

The temperature gradient and hence heat flux at surface is

$$\left.\frac{\partial\theta}{\partial r}\right|_{r=1} = -\sum_{n=1}^{N} b_n \int_o^t \frac{1}{4\tau^2} e^{-\frac{1}{4\tau}} Fn(t - \tau)d\tau \tag{15}$$

some examples of Fn(t − τ) function and the representation of the thermocouple response are:

Case 1 If $\bar{F}n(s) = \frac{1}{s^2 + n^2}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_o^t \frac{1}{2\tau} e^{-\frac{r_1^2}{4\tau}} \frac{1}{n} \sin n(t - \tau) d\tau \tag{16}$$

Case 2 If $\bar{F}n(s) = \frac{s}{s^2 + n^2}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_o^t \frac{1}{2\tau} e^{-\frac{r_1^2}{4\tau}} \cos n(t - \tau) d\tau \tag{17}$$

Case 3 If $\bar{F}n(s) = \frac{1}{(s + a)^n}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_o^t \frac{e^{-\frac{r_1^2}{4\tau}} (t - \tau)^{n-1} e^{-a(t-\tau)}}{2\tau(n - 1)} d\tau \tag{18}$$

Case 4 If $\bar{F}n(s) = s^{-(n+1/2)}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_o^t \frac{e^{-\frac{r_1^2}{4\tau}} 2^n (t - \tau)^{n-1/2}}{2\tau\ 1.3.5...(2n - 1)\sqrt{\pi}} d\tau \tag{19}$$

Case 5 If $\bar{F}n(s) = \frac{1}{s^n}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_o^t e^{-\frac{r_1^2}{4\tau}} \frac{(t - \tau)^{n-1}}{2\ (n - 1)!} d\tau \tag{20}$$

DISCUSSION

In practical application one normally has a measurement of an interior thermocouple response preferably near the heating surface. The temperature measured by the thermocouple can then be fitted with a suitable polynomial given in Eq. (16) through (20). From Chen and Thomsen's (6) work the minimum degree of polynomial N should be about 10 for a transient heating such as in gun barrel and engine. The coefficients b_n obtained from the fitting is then substituted into Eq. (14) for the surface temperature and Eq. (15) for the surface heat flux.

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This report consists of three parts. The first is the analysis for minimization of the temperature distortion due to the thermocouple cavity. The error is minimized or reduced to zero by optimizing the combination of cavity diameter and depth and the thermocouple transport properties and size. The second is the refinement of the presently available computer program for prediction of the surface temperature and heat flux at the
(cont.) over

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1. Heat Conduction
2. Error Analysis
3. Gun Barrel

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HEAT TRANSFER ANALYSIS FOR UNSTEADY HIGH VELOCITY PIPE FLOW

Prepared by: Ching Jen Chen

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